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REVIEW

STUDY ON APPLICATIONS OF LAPLACE TRANSFORMATION: A REVIEW

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ABSTRACT

Background: *Role of mathematics is impeccable in day-to-day life. To justify and in validating their research findings, researchers from life sciences to computer technology uses various Mathematical technique or tool or model. Laplace transform is one of the tool used by scientist and researchers in finding the solution to their problems. In this paper we study the wide range of "Applications of Laplace transformations" in various fields. In this paper 25 research papers were studied and how Laplace transform has been used to solve their research problem is discussed. The current paper gives theory, problem worked on and application of Laplace transform in the research paper. To present a methodical review on applications of Laplace transformation is the objective of this paper. As a significant tool this technique have been used to respond diverse research problem modeled as differential or integral equation. The literature review provides a survey on Laplace transformation technique. The results of numerous studies, allow us to recommend the use of this technique to model their research problem mathematically and to find the solution to the same.*

KEY WORDS
Integral Transform, Laplace transformation, Applications, Analysis

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INTRODUCTION

The significance of transformation technique is it converts the system into an easier form so that solution can be derived from there. For numerous scientific foundations transform theory and techniques are helpful to experts. There are numerous classes of problems that are hard to solve or else algebraically very awkward in their unique portrayals. In applied mathematics, Laplace transformation has key role in concluding the solution guided by complex integral function. Comparing with the variation method of constant and undetermined coefficient this integral transform is simpler in application. In particular the Laplace transformation method is applied in solving the IVP (initial value problem) of nth order linear differential equations with constant coefficients [1].

Pierre-Simon Laplace, a French Mathematician introduced a special type of integral transform in his research later on it was called as Laplace transformation. Oliver Heaviside, a British physicist, developed Laplace transformation systematically. Among the various integral transform it is used mostly. The easiness in understanding and simple in applying is the inspiration behind this transformation technique. In numerous problems Laplace transformation is applied to derive the general solution.

As a significant tool the principal task of Laplace transform is, in establishing the suitable mathematical model for the solution of equations. Laplace transform converts the function f(t) from its time domain to frequency domain F(s). Then inverse Laplace transformation transfers the converted frequency domain F(s) in to time domain. In brief Laplace transform converts differential or integral equations into an algebraic equation. The extensive choice of application makes Laplace transform as a powerful tool in studying the characteristics of engineering problems [1].

Laplace transform is widely used method to solve higher order differential equations. It has many applications in Mathematics, Applied sciences and Engineering. It is also used for calibrating integral, differential, circuit systems, mechanical systems, avionics systems, image processing to say a few. Next section describes the application of Laplace transform in various fields.

STUDY ON APPLICATIONS OF LAPLACE TRANSFORMS

In this section the study about the applications of Laplace transform is discussed. The study is on how this technique is applied in various research problems. Research articles are studied, the theory behind, problem worked on and applications are portrayed. Methodical study is performed to portrait the functionality and the application of Laplace transformation technique.

Simulation of impulse response of electric machines

Electrical machines are often exposed to variety of wave forms which cause some faults in them. The work is done to replicate these impulse responses and know about them. Metwally, 1999 [2] had discussed three methods for deriving transient response of electrical machines are discussed, they are: (i) state space approach (ii) Realization of equivalent circuit (ii) Laplace transformation technique. Laplace transformation is applied to analyze voltage transient.

General non linear modal representation of large scale power system

Hasan Modirshanechi and Naserpari, 2003 [3] introduced and developed a new method called modal series method, which represents non linear system response for even zero input in form of differential equations. It derives and represents the behavior of non linear dynamic systems using non linear modal representation.
In this Laplace transformation is used to solve non linear differential equations.

Three level back to back high voltage direct current converters based on H bridge converters

Modeling of high voltage direct current converters based on H bridge voltage source converters are demonstrated by Siriya Skolthanarat, 2007 [4]. It has more features compared to two level converters. It corrects power system phenomenon like power quality, voltage and first swing stability. Designs of PI type compensators are derived by Laplace transformation.

Analytical techniques for broadband multi electro chemical piezo electric bimorph beams with multi frequency power harvesting

Peter Lloyd woodfield, 2015 [5] derived the multi frequency responses of multi electro chemical piezo electric bimorph beams based on closed form boundary value method from strong form of Hamiltonians principle. Also discussed the Conversion of unused mechanical energy to electrical energy by designing suitable electro mechanical system. Laplace transform is used to design new formulae for power harvesting multi frequency responses for multiple bimorph beams of different types of connections.

Generalized variational principles for heat conduction models based on Laplace transforms

For parabolic and hyperbolic heat conduction equations, Classical variational principle doesn't exist P. Szymczyk, M. Szymczyk, 2015 [6] explained and discussed the principles of those equations. In this classical variational principle is characterized to models like cattaneo-vernotte model, Jeffrey model, two temperature models to say a few. Laplace transformations are used to derive classical variational principles.

Transient analytical solution for the motion of a vibrating cylinder in the stokes regime using Laplace transforms

A stationary Newtonian fluid, a solution for transient decay of moment of vibrational cylinder is studied by Shu-NanLi, Bing-Yang Cao, 2016 [7]. Moment of elastic cylinder is also discussed. In this full expressions for transient terms are specified. It also have applications in viscosity measurements. Laplace transformation is used to derive analytical solutions for moment of elastic cylinder in Newtonian fluid.

Classification of geological structure using ground penetration radar and Laplace transform artificial neural networks

Mikail. F. Lumentat, 2012 [8] described a new type of artificial neurons and neural networks. By using these neural networks and on basis of different types of geological structures, the structure of geological substance is classified. Laplace transform is used instead of artificial weights and in linear activation function of artificial neuron.

SAR image Despeckling based on combination of Laplace mixture distribution with local parameters and Multiscale edge detection in lapped transform domain

The effect of speckle noise on tasks of automatic information extraction and SAR images is studied by D. Hazarika et al., 2016 [9]. And this effect is rectified using Laplace Transform Technique. A new and effective method is developed to SAR image Despeckling. A new type of Laplace orthogonal transform (LOT) is proposed to despeckle SAR images.

Wave propagation and transient response of a fluid filled FGM cylinder with rigid core using the inverse Laplace transform

A study on wave propagation and transient response of fluid filled Functionally Graded Material (FGM) is discussed by K. Daneshjou et al., 2017 [10]. Analytical methodology for deriving transient response of

fluid filled FGM cylindrical shell with a co-axial rigid core. Inverse Laplace transform is used for the study. Derivation of wave propagation, transient response of fluid filled FGM cylinder with rigid core using transform technique carried out.

Analytical solution of Abel integral equation arising in astrophysics via Laplace transforms

Sunil Kumar et al., 2015 [11] discussed an algorithm for Abel integral equation, called as Homotopy Perturbation Transform technique (HPTM). Comparatively it found very simple than other calculation methods. The HPTM is employed to get quick and accurate solutions of singular integral equations of Abel type, linear and non-linear type problems in science and technology. Here the new method HPTM is formed by some modification of Laplace transformation.

Analysis for pressure transient of coalbed methane reservoir based on Laplace transform finite differential method

Lei Wang et al., 2015 [12] have emphasized the mathematical model of coalbed methane based on fractal geometry. Fractal medium are derived from Langmuir isotherm, Adsorption formula, and Fick's diffusion law. Wellbore storage effect and skin effect is considered and mathematical equation is derived. Laplace transform is used as a finite difference method for solving mathematical equations.

Medical application for the flow of carbon-nanotube suspended nanofluids in the presence of convective condition using Laplace transform

Hodasaleh et al., 2013 [13] studied the use of CNT's in Medical sciences; they are used for cancer treatment by sending them to tumor sites by action of waves propagated by walls of artery. The equations for heat flow in CNT's are derived. Laplace transform is used for solving heat transfer equations of CNT's.

A Swiss army knife for finite rate of innovation sampling theory

Ayush Bhanar and Yanina C. Elar, 2016 [14] discussed the General description of a recipe to a wide class of mathematical operations for extension of FRI sampling theory. Development of a broad FRI framework which is applicable to class of transformation such as Laplace, Fourier, Fresnel and few other Transformations is done in this paper. For exact recovery of Dirac impulses from linear measurements in the form of orthogonal projections of streams of Dirac impulses, finite-rate-of-innovation (FRI) sampling theory is use onto the subspace of Fourier-band limited functions. The orthogonal projection of a signal onto the subspace of SAFT- Band limited functions is equivalent to low pass filtering followed by sampling. The representation of FRI signals in the SAFT domain is based this equivalence. Many interesting extensions are done via FRI principles.

Convexly constrained linear inverse problems: iterative least squares and regularization

Ashutosh Sabharwal and Lee C. Potter, 1998 [15] have intensely analyzed the Linear Inverse Problems which are accompanied by convex and closed constraints. Presentation of Conditions for convexly constrained inversion is done. The present analysis proves that this approach extends good to convexly constrained as well as other popular approaches such as L-curve and cross Validation. To regularize constrained inversion, a stopping rule is shown. Also an iteration that converges to the minimum norm least squares solution is presented. To illustrate the proposed algorithm, a constrained Laplace inversion is computed. Extension of Discrepancy principle is done for regularization of ill-posed inverse problems to include convex constraints. Stopping rule uses the knowledge of noise power bound effectively.

New Mathematical nonlinear modular model for switch-mode pulse width modulated converter circuits

Andres Nogueiras et al., 2013 [16] developed a new mathematical model of pulse width modulation (PWM) process to attain behavioral non linear simulation of switch mode power converters. This technique is employed to facilitate the simulations of dc to dc converters and VSI to obtain different PWM techniques. The modeling techniques are very efficient. The obtained data from the simulations are contrasted against data from other authors in previous works with other mathematical simulation models and proved to be valid. To achieve a nonlinear Mathematical Model of PWM pulses, Laplace Transform and Heaviside unitary pulse function is applied.

Reducing torque ripple of brushless dc motor by varying input voltage

Ki-Yong Nam et al., 2006 [17] presented a Method to reduce the torque ripple of brushless direct current motor. In the BLDC motor, the torque ripple is decided by back EMF which is constant in the conduction region of current, torque ripple depends on current ripple. By varying input voltage, torque ripple can be reduced to reduce current ripple. Torque ripple is reduced by ten percent, in the simulation confirmed by the experiment. The torque ripple is not reduced conspicuously. The current ripple is the one which is reduced conspicuously. The produced torque ripple wave form is similar to the back EMF wave form. The period of freewheeling reason in the conduction region is acquired by using Laplace Transformation via circuit analysis.

Electromechanical piezoelectric power harvester frequency response modeling using closed form boundary value methods

Two theoretical studies, CEDRTL and CEDRT have been discussed and compared by Lumentut and Ian M. Howard, 2014 [18]. CEDRTL and CEDRT tend to overlap when the load resistance approaches short circuit. To predict the electromechanical power harvester frequency response, a novel analytical model of a piezoelectric bimorph is presented. Using Laplace Transformation the electromechanical frequency response function with variable load resistance is given. To demonstrate the shifting frequency and amplitude changes due to variable resistance, using NYQUIST plot experimental and CEDRTL model results were very close to each other.

Electrochemical disturbance propagation and oscillation in power systems

A study on electromechanical disturbance propagation and oscillation based on a multi segment uniform change power system is discussed by Delin wang and Xiaoru Wang, 2012 [19]. Same as that of electrochemical wave propagation in a continuum model, the reflection and transmission formulae of electrochemical disturbance is presented. Also, power oscillation and frequency is derived from the view point of electromechanical disturbance propagation. The analytical expressions of Bessel functions reveal that the electromechanical disturbance propagates along the chain power systems. A power system, electromechanical power oscillations is induced by electromechanical disturbance propagation and the oscillation frequency modes could be obtained by a grid structures and parameters. The Machine rotors angle and power increments at all bases are derive using Laplace Transformation under a unit step function disturbance.

Interconnect reliability modeling and analysis for multi-branch interconnect trees

For electro migration reliability analysis in multi branch interconnect trees Hai-Bao Chen et al., 2015 [20] proposed a new modeling and analysis technique with continuous metallization which reflects more practical interconnect structures and writing techniques. Analytical solutions are obtained for each type of the interconnect trees by using Laplace transformation. The exact analytic solutions to stress evolution equations for different terminal wires (3,4 & 5) have been developed. An excellent agreement is shown by the new physics based EM model, for multi branch interconnect trees with the detailed numerical analysis. By De-coupling the individual segments through the proper boundary conditions the new approach solves the stress evolution in a multi branch tree accounting the interactions between different branches.

Neuro computing, supervised learning Laplace transform artificial neural networks an using it for automatic classification of geological structures

A method of learning novel Laplace transform artificial neural network (LTANN) is presented by P. Szymczyk and M. Szymczyk, 2014 [21]. The concept of Laplace transformation is utilized in neural networks. Description of the usage of (LTANN) for searching anomalies in geological structures is explained. This method is based on well known method of supervised learning and it was adopted into a new type of networks (LTANN).

Fast-varying and transient non-linear equations for micro structure fibers

Jing Huang et al., 2017 [22] discussed the fast varying field in micro structure fibers (photonic crystal fibers) is described by the transient non-linear equation. The transient non-linear equation in which the frequency and wave number are functions of time describes the quick evolution of the field in micro structure fibers. By non-linear effect, new frequencies are induced and amplified. The second order differential of the field to transmission distance can't be deleted, which will induce an oscillation along Z in the resonance condition to interpret the principle of photonic crystal fiber metamaterials, this property can

be utilized. Volterra series integration and Laplace transformation are the two methods used to solve the transient non-linear equations.

Novel unilateral NMR sensor for assessing the aging status of silicone rubber insulator

Xu Zheng et al., 2016 [23] proposed a novel unilateral nuclear magnetic resonance (NMR) method to quantify insulator degradation attribute to aging. Aging results in decrease in NMR transverse relaxation time and more serious aging results in a more severe decrease. Values of all SRIs with different service times tend to be nearly the same for sufficiently large depth. Sensor static field and step motor system are employed to obtain respective operation measurements. The Safety of the power grid is threatened by the aging of a silicone rubber insulator. So for taking Precautions against insulation failure, accurate estimation of the insulator aging status is necessary which is found by using the proposed method. By Applying Inverse Laplace Transformation on the filtered transverse relaxation, a decay curves 1-D T2 attribution of each layer is obtained.

A fast algorithm for parabolic PDE-based inverse problems based on Laplace transforms and flexible krylov solvers

Tania Bakhos et al., 2015 [24] proposed a new method to solve parabolic partial differential equations effectively and fast. In their work parameters are estimated for large scale weakly non linear inverse problems for which the governing equations are linear time dependent, parabolic partial differential equations (PDE). Laplace Transformation is applied to solve parabolic partial differential equations. Which improve the storage and computational cost.

Solutions for fractional order electrical circuits in Laplace transform an non standard finite difference method

WK Zahra et al., 2017 [25] studied Fractional linear electrical systems also Introduced new parameters for generalization of RL and RC circuits. The classical electrical system and fractional electrical system are compared. Fractional linear electrical systems are solved by fractional Calculus. This Fractional modeling introduces more accurate representation of real inductor and real Capacitor. Solution for fractional models of RL and RC circuits are derived by Laplace Transform.

Analytical solutions of convection-diffusion problems by combining Laplace transform method and homotopy perturbation

Sumit Gupta et al., 2015 [26] established a method to solve linear and non linear convection diffusion problems arising in physical phenomenon in which energy is transferred due to diffusion and convection. Homotopy perturbation transformation method is introduced which makes solving convection diffusion equations easily. Here HPTM is combination of Laplace transform and Homotopy perturbation.

CONCLUSION

The main purpose of this paper is to give a brief idea about applications of Laplace transforms in various areas and how it is used to solve various types of equations and problems in science, Engineering. In this paper some of the applications of Laplace transforms in various fields are reviewed and explained. This study on Laplace Transformation technique not only offers an insights and exhaustively comprehensive review. It also summarizes the various applications of Laplace transformation technique in various fields, making a general perception of the research progresses, leaning to assist the development and solution to various research problems.

CONFLICT OF INTEREST

There is no conflict of interest.

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