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# PROCESSING LINKED FORMAL FUZZY CONTEXTS USING NON-COMMUTATIVE COMPOSITION

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## ABSTRACT

*This paper focused on analysis of formal fuzzy contexts having common attribute set (Y) as follows: (X, Y, $R_1$) and (Z, Y, $R_2$ ). For this purpose given contexts are linked through composition and further analyzed using projection operator. Since the composition is not commutative i.e Due to this fact the knowledge discovered from the both the compositions i.e.* $(X, Z, R_3 = R_1 * R_2)$ or $(Z, X, R_4 = R_2 * R_1)$ *is compared. One application of the proposed method is also shown for predicting the score of a cricket match.*
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KEY WORDS

*Composed context; Formal Concept Analysis; Formal fuzzy context; Formal fuzzy concept; Knowledge representation.*

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## INTRODUCTION

Knowledge discovery from a given formal context is one of the major concerns for the research communities [1]. For this purpose Wille [2] introduced a mathematical model called as Formal Concept Analysis (FCA). This mathematical model was based on applied lattice theory and its properties. FCA receives the input in form of a binary matrix called as formal context. From the given formal context FCA discovers formal concepts having objects set with their common attributes closed with Galois connection. All the discovered formal concepts can be visualized as a hierarchical ordering in the concept lattice. Several algorithms are proposed for generating the concept lattice with its application in different contexts [3-4]. Further for handling the uncertainty and vagueness in linguistics words like (`young', `tall' etc.) precisely, FCA is linked with fuzzy [5], interval-valued [6-7], and bipolar fuzzy set [8-9]. Recent year's attention has been paid towards linking the formal fuzzy context [10] using composition [11-13], hedges [14] and, shared bond [15]. Motivated from these recent studies, in this paper, we focus on the knowledge processing tasks via linking the fuzzy contexts. Let us suppose a formal fuzzy contexts-: $(X, Y, R_1)$ have the similar attribute set when compare to another formal fuzzy context-$(Z, Y, R_2)$. In this case these two contexts can be linked via composition i.e. $(X, Z, R_3 = R_1 * R_2)$ *or* $(Z, X, R_4 = R_2 * R_1)$ [16]. The composition of contexts contains set of objects (*X*) from the first context and set of attributes (*Z*) from another context. In this case a question arises that how to process the linked fuzzy contexts using composition.

Some attention has been paid towards the knowledge discovery task from the linked fuzzy contexts. In these type of dynamic cases it may not be necessary to work with all formal fuzzy concepts. We need some of the formal concepts which fulfill our requirements based on given objects and attribute set [17-18]. This motivates us to study the operation on fuzzy relations which project it on selected dimensions (like objects and attribute set) for the adequate analysis. In this case the projection of fuzzy relation and its applications with formal context [19-20], formal fuzzy context [21] looks useful. The properties of projection provides us an alternative way to project the computed fuzzy relation on the chosen subset of selected dimensions of the given context [22]. In this paper we try to utilize this vital property of projection operator to handle the composed fuzzy contexts.

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Recently, the analysis of composed context is discussed with an illustrative example [21]. However, the composition is not commutative i.e. $\underset{\sim}{R}_1 * \underset{\sim}{R}_2 \neq \underset{\sim}{R}_2 * \underset{\sim}{R}_1$. Due to that it provides two different fuzzy formal contexts. To reveal some interested concepts through $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$ *or* $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$ is discussed in this paper. Further the comparative study between the knowledge discovered via non--commutative composition is shown with an illustrative example. The proposed method is unique when compare to other available approaches in following aspects:

(1) The proposed method analyze the formal fuzzy contexts using both non--commutative composition i.e. $\underset{\sim}{R}_1 * \underset{\sim}{R}_2 \neq \underset{\sim}{R}_2 * \underset{\sim}{R}_1$ and,

(2) Second it compares the knowledge discovered from both the non-commutative composed contexts with an example.

Other parts of the paper is composed as follows: The related notions of FCA with fuzzy setting is given in section 2. The proposed algorithm and its illustration is shown in section 3 and section 4, respectively. Section 5 contains application of the proposed algorithm to project the cricket score followed by conclusions, acknowledgment and references.

## FORMAL CONCEPT ANALYSIS IN THE FUZZY SETTING

The In this section we recall some basic definition of FCA in the fuzzy setting as given below:

**Definition 1.** (Formal fuzzy context). A fuzzy formal context is a triplet F= (X, Y, ) where X is set of objects, Y is set of attributes and is a fuzzy binary relation on X Y. For any $x \in X$ $y \in Y$, $\underset{\sim}{R}(x, y) \in L$, to which object x has attribute y (L is a support set of some complete residuated lattice L).

**Definition 2.** (Fuzzy set representation of object). Each object x X in a fuzzy formal context K can be represented by a fuzzy set (A) as A(x) = $\{(y_1(\mu_1))$, $(y_2(\mu_2))$, . . ., $(y_m(\mu_m))\}$, where $\{y_1, y_2, \ldots, y_m\}$ is the set of attributes in K and is the membership of x with attribute Y in K. Then A (x) is called the fuzzy representation of object X. Similarly we can define it for attribute B(y).

**Definition 3**. (Residuated Lattice). A residuated lattice L = $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is the finite structure of truth values of object and its properties. L is complete residuated lattice iff:

(1). $(L, \wedge, \vee, 0, 1)$ is a complete lattice.

(2). $(L, \otimes, 1)$ is commutative monoid.

(3) $\otimes$ and $\rightarrow$ are binary operations called multiplication and residuum, respectively i.e. $a \otimes b \leq c$, iff $a \leq b \rightarrow c$ for any *a*, *b*, *c* ∈ *L*.

The operators and are defined distinctly by Lukasiewicz, Gödel, and Goguen independently. as given below:

**Definition 4**. (Formal fuzzy concept). For any L-set A LX of objects and L- set of B LY attributes, we can define a L- set of attributes and L- set of objects as follows:

- $A^{\uparrow}(y) = \wedge_{x \in X}(A(x) \rightarrow \underset{\sim}{R}(x, y))$
- $B^{\downarrow}(y) = \wedge_{y \in Y}(B(y) \rightarrow \underset{\sim}{R}(x, y))$

$A^{\uparrow}(y)$ the truth degree of attribute y is covered by all objects from A and is the truth degree of object x has all attributes from B. Then $A^{\uparrow}(y)$ a pair of (A, B) LX LY is a fuzzy formal concept if and and defines sub-super concept hierarchy as (A1, B1) ≤ (A2, B2) then A1 ⊆ A2 (or B2 ⊇ A1) in L(F).

**Definition 5.** (Fuzzy concept lattice). All the concepts from the formal fuzzy context F are denoted as L(F) and they denies partial order relation: if (A1, B1) ≤ (A2, B2) then A1 ⊆ A2 or B2 ⊇ A1). Thus, (L(F),≤) is a fuzzy concept lattice. It defined by several researchers separately. Together with this ordering in the complete lattice there exist an infimum and a supremum for any two formal concepts given by [2-3]:

- $\wedge_{j \in J}(A_j, B_j) = (\bigcap_{j \in J} A_j, (\bigcup_{j \in J} B_j)^{\downarrow\uparrow})$
- $\vee_{j \in J}(A_j, B_j) = ((\bigcup_{j \in J} A_j)^{\uparrow\downarrow}, \bigcap_{j \in J} B_j)$

Classical logic is special case of FCA with fuzzy setting in which the L-set contains only two elements i.e 1 and 0. The hierarchical order visualization of formal concepts in the concept lattice is an attentive output for the applications of FCA [4]. To get this output from the linked formal fuzzy contexts via their attribute set is major concern of researchers. The issue includes the connection of these two context and their adequate analysis. For this purpose in this paper a method is proposed to link these contexts based on composition. Further the composed context is analyzed with projection on objects and attribute set. The followings are given in the next section.

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## PROPOSED METHOD

Let us suppose we have given a formal fuzzy context-F = : $(X, Y, \underset{\sim}{R})$. In any given fuzzy context following possible conditions may exists as shown in **[Table-1]**. Complete (or incomplete) represents that the information is available (or not-available) for the given sets [26--27]. To link the data set we should have the complete information about the given contexts [10-11, 28]. Then the given fuzzy contexts i.e. $(X, Y, \underset{\sim}{R}_1)$ and $(Z, Y, \underset{\sim}{R}_2)$ can be linked as follows:

**Table 1: Some of the well-known conditions in a given formal fuzzy context**

| Conditions | Object | Attribute | Fuzzy Relation |
|---|---|---|---|
| a | Complete | Complete | Incomplete |
| b | Incomplete | Complete | Complete |
| c | Complete | Incomplete | Complete |
| d | Complete | Complete | Complete |
| e | Incomplete | Incomplete | Incomplete |

**Step 1.** The given fuzzy contexts can be linked via composition $\underset{\sim}{R}_1 \times \underset{\sim}{R}_2 = \underset{\sim}{R}_3$ or $\underset{\sim}{R}_2 \times \underset{\sim}{R}_1 = \underset{\sim}{R}_4$. The composed fuzzy contexts $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$ *or* $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$ includes objects from first context and attributes from second. In this scenario it can be process as follows:

**Step 2.** The fuzzy relation for the given composed context $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$ can be projected on objects set-($X$) as follows: $\prod_{R3}(X) = \{(x), \max_z \mu_{R3}(x, z_k) \in X \times Z\}$. This provides the maximum degree of membership value from the tuples ($x$, $z_k$) in the given fuzzy formal context. Subsequently we can find the projection on attribute set $\prod_{R3}(Z) = \{(z), \max_X \mu_{R3}(x_i, z) \in X \times Z\}$. These projection on object and attribute set provides two fuzzy contexts based on object and attribute set of context $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$.

**Step 3.** Similarly second composed fuzzy context $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$ can be projected on objects and attribute set for knowledge discovery task as per step 2. This will also provide two distinct fuzzy context can be received based on projection. Further we can generate the formal concepts to reveal the interested pattern in the data set.

**Step 4.** The composition of fuzzy contexts is not commutative. Due to this fact we can compare the knowledge discovered from both the composed contexts for adequate analysis.

The above steps are shown as pseudo code in the **[Table-2]**. The proposed algorithm receives input as fuzzy contexts and link them using composition of matrix code as shown in steps 2 to 4. Then it computes the projection on objects using the steps 5 and 6. To generate the fuzzy concept lattice based on object concept as given in step 7. Similarly the proposed algorithm provides a projection on attribute set using the steps 9 and 10. To generate the fuzzy concepts for the attribute context at step 11. In the end the proposed algorithm provides two fuzzy concept lattice structure based on objects and attribute set from the composed context $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$. Similarly the proposed method provide two fuzzy concept lattice for another (non -commutative) composition- $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$. In this way the proposed method provides an alternative way to analyze the linked formal contexts based on their projection rather than one independent context. Moreover, it optimizes the possibility to find some interested formal concepts in less computational complexity.

**Complexity:** Let us suppose, number of objects in first fuzzy context (|$X$|) = $n$, and number of objects in second fuzzy context (|$Z$|)=$k$ where number of attributes in both of the context (|$Y$|) = $m$. Now suppose max {$n$, $m$, $k$}=$n$. Then in the computing the composition of fuzzy contexts will takes O($n^3$) time. Further, the proposed method finds the projection on objects and attributes set which takes complexity O($n^2$). Hence the overall computation takes O($n^3 + n^2$) time.

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**Table: 2. Proposed Algorithm for processing the linked formal fuzzy contexts**

**Input : Given two formal fuzzy contexts($X$,$Y$, $R_1$) and ($Z$,$Y$, $R_2$)**
**Output: Formal fuzzy concepts based on objects and attribute set**

(1) Enter the matrix ($X$,$Y$, $R_1$) and ($Z$,$Y$, $R_2$)
(2) Compose the matrix $R_1 * R_2$ as follows:

```
for (i=0;i<n;i++)
  for(j=0;j<m;j++)
      z[i][j]=0;
        for (k=0;k< max(n,m);k++)
          z[i][j]+= x[i][k]*y[k][j].
```

(3) Return the composed matrix $(X, Z, R_3 = \underset{\sim}{R}_1 * \underset{\sim}{R}_2)$
(4) End the for loop
(5) Project the composed context based on object set
for (i=0;i<n;i++)

$$\prod_{R3}(X) = \{(x), \max_z \mu_{R3}(x, z_k) \in X \times Z\}$$

(6) Return the context obtain using projection on object set.
(7) Generate the formal fuzzy concept lattice for this context.
(8) End for loop.
(9) Similarly project the composed context based on object set
for (k=0;j<k;k++)

$$\prod_{R3}(Z) = \{(z), \max_X \mu_{R3}(x_i, z) \in X \times Z\}$$

(10) Return the context obtain using projection on object set.
(11) Generate the formal fuzzy concept lattice for this context.
(12) End for loop.
(13) Similarly other composed context $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$ can be processed.

## KNOWLEDGE DISCOVERY USING COMPOSITION OF FUZZY CONTEXTS

In this section we illustrate the proposed algorithm through an example. Let us suppose a company want to analyze the suitability of some candidate to offer the employments-Domestichelper, Waiter, Accountant, Carsalesman based on the following knowledge-Computer Science, Accounting, Mechanical, Cooking as represented in **[Table-3]**. [11-12]. In the same time company receives some CV from the students--$C_1$, $C_2$, $C_3$, $C_4$, $C_5$ based on their knowledge-Computer Science, Accounting, Mechanical, Cooking as shown in **[Table-4]**. Now a problem arises with company that how to discover the knowledge from these given contexts to offer the employments.

**Table: 3. Different knowledge required for the employment in a company i.e.** $(X, Y, \underset{\sim}{R}_1)$

| Employment | Computer Science | Accounting | Mechanical | Cooking |
|---|---|---|---|---|
| Domestic Helper | 0.1 | 0.3 | 0.1 | 1.0 |
| Waiter | 0.0 | 0.4 | 0.0 | 0.7 |
| Accountant | 0.9 | 1.0 | 0.0 | 0.0 |
| Car Salesman | 0.5 | 0.7 | 0.9 | 0.0 |

**Table: 4. CV received from the candidates based on the given knowledge i.e** $(Z, Y, \underset{\sim}{R}_1)$

| Employment | Computer Science | Accounting | Mechanical | Cooking |
|---|---|---|---|---|
| $C_1$ | 0.5 | 0.8 | 0.3 | 0.6 |
| $C_2$ | 0.2 | 0.5 | 0.1 | 1.0 |
| $C_3$ | 0.0 | 0.2 | 0.0 | 0.3 |
| $C_4$ | 0.9 | 0.4 | 0.1 | 0.5 |
| $C_5$ | 0.7 | 0.5 | 0.2 | 0.1 |

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**Table :5.Composition of Table 3 and Table 4 i.e.** $(X, Z, R_3 = \tilde{R}_1 * \tilde{R}_2)$

| $R_1$*$R_2$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| Domestic Helper | 0.6 | 1.0 | 0.3 | 0.5 | 0.1 |
| Waiter | 0.9 | 1.0 | 0.6 | 0.8 | 0.4 |
| Accountant | 0.6 | 0.3 | 0.1 | 0.4 | 0.5 |
| Car Salesman | 0.4 | 0.2 | 0.5 | 0.2 | 0.3 |

To solve this problem we can link these two context using composition ($X$, $Z$, $R_3$=$R_1$*$R_2$) as shown in **[Table-5]**. From the composed contexts shown in **[Table-5]** we can generate the fuzzy formal concepts to analyze the company and candidate requirements which are as follows:

(1). Company requires a suitable candidate based on the advertisement shown in **[Table-3]**.
(2). Candidate requires a suitable job based on their knowledge shown in **[Table-4]**.

To solve the above mentioned problems, we require some of the formal concepts based on objects--(employments ($X$)) and attributes--(Candidates (Z)) set. This can be achieved through the proposed method. **[Table-5]** can be projected based on set of objects--(employment-X) as follows:
(Domestic helper, $C_2$)=1.0,
(Waiter, $C_2$)=1.0,
(Accountant, $C_1$)=0.6,
(Carsalesman, $C_3$)=0.5.
All computed fuzzy relations are shown in **[Table-6]**.

**Table: 6. Projection of Table 5 on object set i.e. employment (X)**

| Employment | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| Domestic Helper | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| Waiter | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| Accountant | 0.6 | 0.0 | 0.0 | 0.0 | 0.0 |
| Car Salesman | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |

The generated formal fuzzy concepts from **[Table-6]** are:

1. $\{\{1.0/\text{Domestic helper} + 1.0/\text{Waiter} + 1.0/\text{Accountant} + 1.0/\text{Carsalesman}\}, \varnothing\}$,
2. $\{\{1.0/\text{Accountant}\}, 0.6/C_1\}$,
3. $\{\{1.0/\text{Domestichelper} + 1.0/\text{Waiter}\}, 1.0/C_2\}$,
4. $\{\{1.0/\text{Carsalesman}\}, 0.5/C_3\}$,
5. $\{\varnothing, 1.0/C_1 + 1.0/C_2 + 1.0/C_3 + 1.0/C_4 + 1.0/C_5\}$.

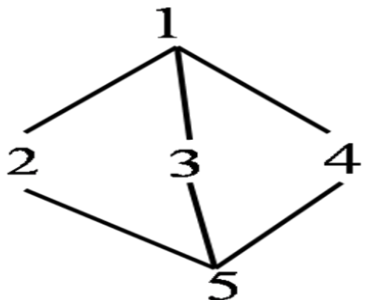


**Fig: 1. Fuzzy concept lattice generated for Table-6**

All the above generated concepts are shown in **Figure-1**. From that following information can be extracted:

A. Concept 2 includes that the candidate-$C_1$ is suitable for Accountant.
B. Concept 3 includes that candidate- $C_2$ is suitable for Domestichelper or Waiter.
C. Similarly, concept 4 includes that candidate-$C_3$ is suitable for Carsalesman.

The above information is helpful for the company.
Similarly, projection on attributes (candidate-Z) of the context shown in [Table-5] can be computed as follows:
(Waiter, $C_1$)= 0.9,
(Domestic helper, $C_2$)=1.0,
(Waiter, $C_2$)=1.0,
(Waiter, $C_3$)=0.6,
(Waiter, $C_4$)=0.8,
(Accountant, $C_5$)=0.5.
All the computed fuzzy relations using the projection on attributes is shown in [Table-7].

**Table:7. Projection of Table 5 on attribute set i.e. candidate (Z)**

| Employment | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| Domestic Helper | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| Waiter | 0.9 | 1.0 | 0.6 | 0.8 | 0.0 |
| Accountant | 0.0 | 0.0 | 0.0 | 0.0 | 0.5 |
| Car Salesman | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

The generated formal fuzzy concepts from [Table-7] are:

1. $\{\{1.0/\text{Domestic helper}+1.0/\text{Waiter}+1.0/\text{Accountant}+1.0/\text{Carsalesman}\},\varnothing\}$,
2. $\{\{1.0/\text{Domestichelper} + 1.0/\text{Waiter}\},\{1.0/C_2\}\}$,
3. $\{\{1.0/\text{Waiter}\},\{1.0/C_1+1.0/C_2+1.0/C_3+1.0/C_4\}\}$,
4. $\{\{1.0/\text{Accountant}\},\{0.5/C_3\}\}$,
5. $\{\varnothing,\{1.0/C_1+1.0/C_2+1.0/C_3+1.0/C_4+1.0/C_5\}\}$.

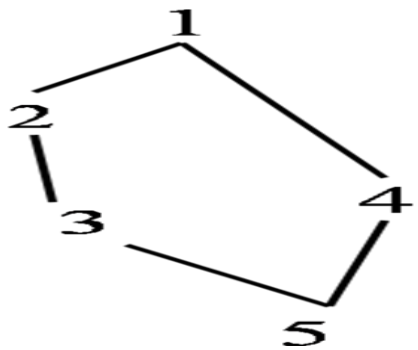


**Fig: 2. Fuzzy concept lattice generated for Table 7**

The formal concept generated from [Table-7] is shown in Figure-2From which following information can be extracted:

(A) Concept 2 includes that candidate-$C_2$ is suitable for Domestic helper or Waiter.
(B) Concept 3 includes that almost each candidates are suitable for Waiter. Among them candidate-$C_2$ is highly eligible.
(C) Similarly, concept 4 includes that candidate-$C_5$ is suitable for Accountant.

The above information is useful for the candidate. The obtained conclusions from Figures-1 and 2 shown in [Table-8].

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**Table: 8. Conclusions obtained from Figure 1and Figure-2**

| Candidate | $C_1$ |
|---|---|
| $C_1$ | Accountant or Waiter |
| C2 | Domestichelper or Waiter |
| C3 | Car Salesman |
| $C_4$ | Not Specific for any Job |
| $C_5$ | Accountant |

Similarly we can analyze another non-commutative composition of the given contexts i.e. $R_2$ *$R_1$ as shown in **[Table-9]**.

**Table:9. Composition of Table 3 and Table 4 i.e.** $(Z, X, R_4 = \underset{\sim}{R}_2 * \underset{\sim}{R}_1)$

| $R_2$*$R_1$ | Domestic helper | Waiter | Accountant | Car salesman |
|---|---|---|---|---|
| $C_1$ | 0.5 | 0.5 | 0.4 | 0.4 |
| $C_2$ | 0.8 | 0.7 | 0.0 | 0.0 |
| $C_3$ | 0.3 | 0.2 | 0.2 | 0.7 |
| $C_4$ | 0.2 | 0.1 | 0.5 | 0.5 |
| $C_5$ | 0.4 | 0.3 | 0.8 | 0.8 |

The projection of composed fuzzy context shown in **[Table-9]** on objects (i.e. candidate) is as follows:
($C_1$, Domestichelper)= 0.5,
($C_1$, Waiter)= 0.5,
($C_2$, Domestichelper)= 0.8,
($C_3$, Carsalesman)= 0.7,
($C_4$, Accountant)= 0.5,
($C_4$, Carsalesman)= 0.5,
($C_5$, Accountant)= 0.8,
($C_5$, Carsalesman)= 0.8.
These relations are shown in **[Table-10]**.

**Table 10. Projection of Table 9 on objects set i.e. Candidate (Z)**

| $R_2$*$R_1$ | Domestic helper | Waiter | Accountant | Car salesman |
|---|---|---|---|---|
| $C_1$ | 0.5 | 0.5 | 0.0 | 0.0 |
| $C_2$ | 0.8 | 0.0 | 0.0 | 0.0 |
| $C_3$ | 0.0 | 0.0 | 0.0 | 0.7 |
| $C_4$ | 0.0 | 0.0 | 0.5 | 0.5 |
| $C_5$ | 0.0 | 0.0 | 0.8 | 0.8 |

The generated formal fuzzy concepts from **[Table-10]** are:

1. $\{\varnothing, \{1.0/C_1 + 1.0/C_2 + 1.0/C_3 + 1.0/C_4 + 1.0/C_5\}\}$.
2. $\{0.5/C_1 + 0.5/C_2$, 1.0/Domestichelper },
3. $\{0.5/C_1 + 0.5/C_2$, 1.0/Domestichelper+1.0/Waiter },
4. $\{0.5/C_4 + 0.8/C_5$, 1.0/Accountant+1.0/Carsalesman},
5. $\{0.7/C_3 + 0.5/C_4 + 0.8/C_5$, 1.0/Accountant},
6. {{1.0/ Domestic helper + 1.0/Waiter+1.0/Accountant+1.0/Carsalesman}, $\varnothing$}.

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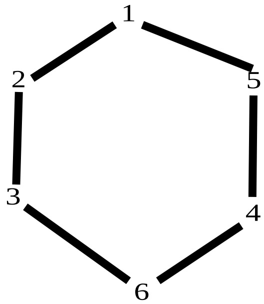


**Fig: 3. Fuzzy concept lattice generated for Table 10**

The concepts generated from the **[Table-10]** are shown in the concept lattice of Fig. 3. From that following information can be extracted:

(A) Concept 2 includes that the for the employment of Domestichelper candidates $C_1$ and $C_2$ are suitable .
(B) Concept 3 includes that for the employment of Domestichelper or Waiter candidate $C_1$ is suitable.
(C) Concept 4 includes that for the employment of Accountant or Carsalesman candidates $C_4$ and $C_5$ are suitable .
(D) Concept 5 includes that for the employment of Carsalesman candidates-$C_3$, $C_4$ and $C_5$ are suitable.

Above conclusions are helpful for the candidates.

Similarly, the projection of composed context shown in **[Table-9]** can be computed on attributes (i.e. employment) as follows:

($C_1$, Domestichelper)= 0.8,
($C_2$, Waiter)= 0.7,
($C_3$, Accountant)= 0.8,
($C_4$, Carsalesman)= 0.8.

These computed relations are represented in the **[Table-11]**.

**Table: 11. Projection of Table 9 on attribute set i.e. Employment (X)**

| $R_2$*$R_1$ | Domestic helper | Waiter | Accountant | Car salesman |
|---|---|---|---|---|
| $C_1$ | 0.0 | 0.0 | 0.0 | 0.0 |
| $C_2$ | 0.8 | 0.7 | 0.0 | 0.0 |
| $C_3$ | 0.0 | 0.0 | 0.0 | 0.0 |
| $C_4$ | 0.0 | 0.0 | 0.0 | 0.0 |
| $C_5$ | 0.0 | 0.0 | 0.8 | 0.8 |

The generated formal fuzzy concepts from **[Table-11]** are:

1. $\{\varnothing,\{1.0/C_1+1.0/C_2+1.0/C_3+1.0/C_4+1.0/C_5\}\}$.
2. $\{0.8/C_2$, 1.0/Domestichelper },
3. $\{0.7/C_2$, 1.0/Domestichelper+1.0/Waiter },
4. $\{0.8/C_5$, 1.0/Accountant+1.0/Carsalesman},
5. {{1.0/Domestic helper+1.0/Waiter+1.0/Accountant+1.0/Carsalesman},$\varnothing$}.

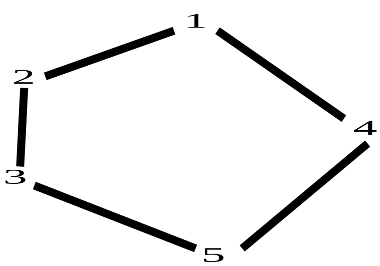


**Fig:4. Fuzzy concept lattice generated for Table 11**

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Concept lattice builds by concept generated from **[Table-11]** is shown in Fig. 4. From that following information's can be concluded:

(A) Concept 2 includes that for the employment of Domestichelper candidates-$C_2$ is suitable.

(A) Concept 3 includes that for employment of Domestichelper or Waiter candidates-$C_2$ is suitable.

(C) Concept 4 includes that for employment of Accountant or Carsalesman candidates-$C_5$ is suitable.

The above information is useful for the company for shortlisting the CV. The obtained conclusions from **Fig. 3 and Fig. 4** are shown in **[Table-12]**.

**Table 12. Conclusions obtained from Fig. 3 and Fig. 4**

| Candidate | $C_1$ |
|---|---|
| $C_1$ | Domestichelper or Waiter |
| C2 | Domestichelper or Waiter |
| C3 | Car Salesman |
| $C_4$ | Accountant or Carsalesman |
| $C_5$ | Accountant or Carsalesman |

Now we need to compare the knowledge discovered from the both the composition shown in **[Table-9]** and **[Table-12]**. For this purpose the comparative study via knowledge discovered from both the composition is shown in **[Table-13]**. .

**Table: 13. Conclusions obtained from Fig. 3 and Fig. 4**

| Candidates | Conclusion from $R_1$*$R_2$ | Conclusion from $R_2$*$R_1$ | Comparative knowledge discovered |
|---|---|---|---|
| $C_1$ | Accountant or Waiter | Domestichelper or Waiter | Waiter |
| C2 | Domestichelper or Waiter | Domestichelper or Waiter | Domestichelper or Waiter |
| C3 | Car Salesman | Car Salesman | Car Salesman |
| $C_4$ | Job cannot be derived | Accountant or Carsalesman | Accountant or Carsalesman |
| $C_5$ | Accountant | Accountant or Carsalesman | Accountant |

From **Table 13** we can derive the following informations:

(A) Candidate $C_1$ is suitable for Accountant, Domestichelper or Waiter.

(B) Candidate $C_2$ is suitable for Domestichelper or Waiter.

(C) Candidate $C_3$ is suitable for Carsalesman.

(D) Candidate $C_4$ is suitable for Accountant or Carsalesman.

(E) Candidate C5 is suitable for Accountant or Carsalesman.

The above information shown in Table 13 accelerates the company work while choosing the candidates for the given employment. Similarly the proposed method can be applied in in searching advanced query in mobile cloud computing [29-30], fuzzy homomorphism [31], keyword extraction [32], and processing the link data bases (Algal Image Database of India (AIDI) available online at *http://indianalgae.co.in*) [33]. In the next section one real life application of the proposed method is demonstrated.

## APPLICATION OF THE PROPOSED METHOD FOR PROJECTING THE CRICKET SCORE

The proposed method can be applied for predicting the score of a cricket or Football games as well. For example: Suppose we want to measure the relationship between condition of pitch and runs scored based on following parameters [13]:

(1). Conditions of Pitches: Good Wicket, Fair Wicket, Sporting Wicket, Green Wicket, Crumbling Wicket, Rough Wicket.

(2). Speed of Bowling: Fast, Medium, Spin, and

(3) Runs Scored: Low Score, Average Score, High Score.

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The relation between first two parameters i.e. conditions of pitches and speed of bowling has been shown in [Table-14]. Similarly the [Table-15] shows a fuzzy relation based on condition of pitches and runs scored.

**Table: 14. A Fuzzy context based on speed of bowling and condition of pitches ($R_1$)**

| | Fast | Medium | Spin |
|---|---|---|---|
| Good Wicket | 0.6 | 0.8 | 0.7 |
| Fair Wicket | 0.5 | 0.6 | 0.8 |
| Sporting Wicket | 0.4 | 0.9 | 0.6 |
| Green Wicket | 0.1 | 0.2 | 0.7 |
| Crumbling Wicket | 0.9 | 0.1 | 0.1 |
| Rough Wicket | 0.5 | 0.6 | 0.2 |

**Table:15. A Fuzzy context based on speed of bowling and run scored ($R_2$)**

| | Low Score | Average Score | High Score |
|---|---|---|---|
| Good Wicket | 0.4 | 0.8 | 0.7 |
| Fair Wicket | 0.3 | 0.8 | 0.8 |
| Sporting Wicket | 0.2 | 0.7 | 0.8 |
| Green Wicket | 0.8 | 0.6 | 0.4 |
| Crumbling Wicket | 0.9 | 0.1 | 0.1 |
| Rough Wicket | 0.5 | 0.6 | 0.2 |

The contexts shown in [Table-14] and [Table-15] are related via similar attribute set i.e. conditions of pitches. In this case how to analyze these fuzzy contexts and generate the fuzzy formal concept is major concerns. This problem can be solved using the proposed method in this paper using the composition as shown in [Table-16]. This composed context (shown in [Table-16]) can be projected based on runs scored and speed of bowling as shown in [Table-17] and [Table-18] respectively.

**Table 16. Composition of Fuzzy context shown in Table 14 and Table 15**

| | Low Score | Average Score | High Score |
|---|---|---|---|
| Fast | 0.7 | 0.6 | 0.6 |
| Medium | 0.6 | 0.8 | 0.8 |
| Spin | 0.7 | 0.8 | 0.8 |

**Table 17. Projection of context shown in Table 16 on speed of bowling**

| | Low Score | Average Score | High Score |
|---|---|---|---|
| Fast | 0.7 | 0.0 | 0.0 |
| Medium | 0.0 | 0.8 | 0.8 |
| Spin | 0.0 | 0.8 | 0.8 |

**Table 18. Projection of context shown in Table 16 on runs scored**

| | Low Score | Average Score | High Score |
|---|---|---|---|
| Fast | 0.7 | 0.0 | 0.0 |
| Medium | 0.0 | 0.8 | 0.8 |
| Spin | 0.7 | 0.8 | 0.8 |

**From Table 17 and Table 18** we can find following information:

1. If speed of bowling is fast or spin then runs scored will be low.\
2. If speed of bowling is medium or spin then runs scored is average or high.

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Similarly the proposed method can be applied in several games like football match for prediction of number of goals. In future work will be focused on extending the proposed method to interval-valued formal fuzzy context [7-9] with its application.

## CONCLUSION

This paper discussed a method to discover the knowledge from the linked contexts i.e.-(X, Y, R1), (Z, Y, R2) using composition i.e. R1* R2 or R1* R2 with complexity O(n3+n2). It is well known that the composition is not commutative i.e.  $. Hence the knowledge discovered from both the composed contexts are compared using their projection on objects and attributes set as shown in **[Table-13]**. The analysis derived from the proposed method is extensive and adequate with [21].

## CONFLICT OF INTEREST

Author accepts that there is no conflict of interests.

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