

FAST ANISOTROPIC FILTER BASED EM WITH SPATIAL INFORMATION FOR SEGMENTATION OF NOISY IMAGES

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ABSTRACT

This paper proposes a new method of Gaussian Mixture Model (GMM) based segmentation of noisy images. Gaussian Mixture model is the most widely utilized methods for segmentation of images and the parameters of GMM are calculated using the Expectation Maximization algorithm (EM). But the conventional Gaussian Mixture Model with Expectation Maximization gives poor segmentation accuracy for images with noise content. This is due to the fact that GMM considers each pixel as an independent data and calculates the distribution. It does not include the spatial data of the surrounding pixels. Many segmentation algorithms suffer from over-smoothness for segmentation and they fail to preserve the image details. Anisotropic diffusion filtering is a successful method which overcomes these undesirable effects in segmentation. In the proposed method, firstly the anisotropic diffusion filtering is applied to the image corrupted by noise to incorporate the local information. Secondly, the EM algorithm is enhanced by incorporating spatial information in the posterior probability which makes the convergence of EM algorithm also faster. The quantitative results obtained by applying the proposed anisotropic filter based EM with spatial information method on synthetic images and simulated brain images and comparison with the other methods demonstrate that the proposed method outperforms GMM by 26% and the recent method in literature by around 1%. The execution time is less compared to the other methods.

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INTRODUCTION

Segmentation of images intends to split the image into different parts having similar features. Segmentation finds its applications in wide areas including medical diagnosis, remote sensing and military applications. The most popular methods available for image segmentation are model based segmentation and clustering methods. Among the clustering methods, the prevalent method for image segmentation is Fuzzy c-means algorithm (FCM) [1,2]. Statistical methods such as Gaussian Mixture Model (GMM) for segmentation is another widespread method where the parameters of GMM are estimated using Expectation Maximization (EM) algorithm [3,4]. The conventional FCM and GMM work well on images without noise, but they fail to segment images corrupted by noise since they do not consider spatial related information in an image.

To integrate the spatial related information in image, the methods in the literature provide several techniques for both FCM and GMM. Blekas et al. proposed Markov Random Field priors [5] for Image segmentation to incorporate the spatial interaction among neighboring pixels. Thanh et al. [6] proposed a non-symmetric mixture model-Student's t-distribution and EM is adapted to estimate the model parameters. A self adaptive GMM [7] and neighborhood weighted GMM [8] are proposed to extend the standard GMM for suitable applications. Likewise, among the FCM based segmentation methods, Maoguo et al. [9] proposed a tradeoff weighted fuzzy factor and kernel metric to incorporate the spatial information. An RBF kernel based intuitionistic FCM replacing the Euclidean distance metric [10] is proposed to segment noisy medical images. Stelios et al. [11] proposed Fuzzy Local Information C Means Clustering (FLICM) in which he suggested the method of incorporating both spatial information and intensity information in a fuzzy manner.

Perona and Malik [12] introduced the anisotropic diffusion filter which provides an effective way of denoising images. Anisotropic diffusion based image enhancement methods are proposed in [13-17] and it is established that it is an efficient way to incorporate the local information while segmentation of noisy images. Based on the above considerations, in the proposed method, first the anisotropic diffusion filter output of the noisy image is obtained.

Then GMM based modified EM segmentation is done where the EM algorithm is enhanced by incorporating spatial information in the posterior probability. By modifying the posterior probability, the EM algorithm converges faster.

The remainder of the paper is structured as follows: Gaussian Mixture Model is detailed, the anisotropic diffusion filter is described, the details of the proposed anisotropic filter based EM with the spatial information method are explained, the experimental solutions are presented and then conclusions are given.

Gaussian mixture model and expectation maximization algorithm

The Gaussian distribution of the variable y is modelled as

$$G(y|\mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} \exp\left\{-\frac{1}{2\sigma^2}(y - \mu)^2\right\} \quad (1)$$

where μ is the mean and σ^2 is the variance.

The likelihood function for the Gaussian distribution is

$$p(x/\mu, \sigma^2) = \prod_{n=1}^N G(x_n/\mu, \sigma^2) \quad (2)$$

The Gaussian mixture distribution can be composed as a linear superposition of M Gaussian densities of the form

$$p(x) = \sum_{l=1}^M \pi_l G(x/\mu_l, \sigma_l^2) \quad (3)$$

where the variable l denotes the class and M denotes the number of classes.

Each Gaussian density $G(x/\mu_l, \sigma_l^2)$ is called a component of the mixture with its own mean μ_l and covariance σ_l^2 . The parameter π_l are called the mixing coefficients.

$$\sum_{l=1}^M \pi_l = 1 \text{ and } 0 \leq \pi_l \leq 1 \quad (4)$$

From Bayes' theorem, the posterior probabilities $p(l/x)$ are given by

$$\gamma_l(x) = p(l/x) = \frac{\pi_l G(x|\mu_l, \sigma_l^2)}{\sum_{l=1}^M \pi_l G(x|\mu_l, \sigma_l^2)} \quad (5)$$

From Equation 2, The log of the likelihood function is given by

$$\ln p(x/\mu, \sigma^2) = \sum_{n=1}^N \ln\{\sum_{l=1}^M \pi_l N(x_n|\mu_l, \sigma_l^2)\} \quad (6)$$

For maximizing the log likelihood, the derivatives of equation 6 with respect to μ_l, σ_l^2 and π_l are set to zero.

$$N_l = \sum_{n=1}^N \gamma_l(x) \quad (7)$$

The parameters are obtained as

$$\mu_l = \frac{1}{N_l} \sum_{n=1}^N \gamma_l(x) x_n \quad (8)$$

$$\sigma_l^2 = \frac{1}{N_l} \sum_{n=1}^N \gamma_l(x) (x_n - \mu_l)(x_n - \mu_l)^T \quad (9)$$

$$\pi_l = \frac{N_l}{N} \quad (10)$$

The EM algorithm is explained in the next steps.

The object is to maximize the likelihood function with regard to the parameters – mean, variance and mixing coefficient.

Step 1: The values of means μ_l , variances σ_l^2 and mixing coefficients π_l for all the classes are initialized

the initial value of the log likelihood is evaluated.

Step 2:E Step;The posterior probabilities are estimated using the current parameter values. (Equation 5).

Step 3: M Step;The parameters means, variances, and mixing coefficients for all the classes are re-calculated using the current posterior probabilities. (Equations 7-10).

Step 4: The Log likelihood is evaluated using Equation 6 and the convergence criterion is checked. If it is not satisfied, return to step 2.

The drawback existing in Gaussian Mixture Model based segmentation is that it considers each pixel as an independent data and calculates the distribution. It does not include the spatial data of the surrounding pixels. Hence the method is sensitive to noise and illumination.

Anisotropic diffusion filtering

The conventional low pass filtering and linear diffusion of an image can be obtained by convolution of the original image with a Gaussian kernel. In this method, noise can be eliminated but the edges are blurred. In anisotropic diffusion, by the proper choice of conduction coefficient, the edges are enhanced and also the noise is eliminated. The equation for anisotropic diffusion is stated as

$$\frac{\partial G(x,y,t)}{\partial t} = \text{div}[g(|\nabla G(x,y,t)|)\nabla G(x,y,t)] \quad (11)$$

where t is the time parameter, $G(x,y,0)$ is the input Gray scale image, $\nabla G(x,y,t)$ is the gradient of the image at given time t , and $h(\cdot)$ represents the conductance function. This function is selected such as to satisfy $\lim_{x \rightarrow 0} h(x) = 1$ and $\lim_{x \rightarrow \infty} h(x) = 0$. Hence, across the uniform regions, the diffusion is maximal and the diffusion is zero at the edges.

The conductance functions which are proposed in [13] are

$$h_1(x) = \exp\left[-\left(\frac{x}{T}\right)^2\right] \quad (12)$$

$$\text{and } h_2(x) = \frac{1}{1 + \left(\frac{x}{T}\right)^2} \quad (13)$$

The rate of diffusion is controlled by the gradient magnitude threshold parameter T . It acts as a threshold between the image gradients contributed by noise and those contributed by edges.

The anisotropic diffusion in discrete form is represented as

$$G_{t+1}(p) = G_t(p) + \frac{\lambda}{|\eta_p|} \sum_{s \in \eta_p} h_k(|\Delta G_{p,s}|) \Delta G_{p,s} \quad (14)$$

Where G is the input digital image, p denotes the pixel position in the image, t denotes the iteration step, h is the conductance function and K represents the gradient threshold parameter. The rate of diffusion is given by the constant λ and it lies in the interval $[0,1]$. η_p represents the 8-pixel neighborhood of pixel s . The gradient is calculated as the difference between neighboring pixels for all the directions.

$$\nabla G_{p,s} = G_t(s) - G_t(p), \quad s \in \eta_p \quad (15)$$

PROPOSED METHOD

The proposed method can be summarized as follows.

A. Apply Anisotropic Diffusion filter to the input noisy image.

Step 1: Set the number of iterations, gradient threshold parameter and the Constant λ .

Step 2: Obtain the gradient of the image in 8 directions. (Equation 15)

Step 3: Determine the conductance functions for the gradients. (Equation 13)

Step 4: Implement the anisotropic diffusion for the image. (Equation 14)

Step 5: Repeat steps 2 through 4 for the total number of iterations.

B. Apply Fast modified EM to the anisotropic filtered image

Step 1: Initialize the number of classes and convergence value of log likelihood and the parameters – mean, variance and mixing coefficient.

Step 2: Evaluate the initial log likelihood.

Step 3: Expectation Step: Estimate the posterior probabilities using current parameters. (Equation 9)

Step 4: Apply arithmetic mean filter to the posterior probabilities to incorporate the spatial information about the neighboring pixels.

Step 5: Maximization Step: Re-estimate the parameters – mean, variance and mixing coefficient using current posterior probabilities. (Equations 7-10)

Step 6: Calculate the log likelihood (Equation 6) and verify convergence. If the log likelihood value does not converge, return to step 2.

Step 7: Obtain the final model and perform segmentation based on the model.

Experimental results

The segmentation results of the method on synthetic images as well as simulated data sets is presented. The processor specification is Intel Core 2 Duo CPU @2.93GHz and the RAM specification is 4.0 GB. The algorithm is implemented with MATLAB.

Experiments on the synthetic images

The method is examined on three test synthetic images. Images similar to those used in [9] are employed. The first image with 128 x 128 pixels comprises of two classes with two intensity values - 20 and 120 as shown in **Figure-1 (a)**. The other two images are of resolution 244 x 244 and 256 x 256 pixels respectively, and shown in **Figure-2 (a)** and **Figure-3 (a)**, for which the number of clusters is 4. For the Expectation Maximization algorithm, the means of the classes are initialized by the means obtained from the K-Means segmentation of the noisy images. The threshold for the log likelihood is set as 0.01.

The parameters - Number of iterations, gradient threshold parameter and the Constant λ for anisotropic diffusion filtering are set as 25, 25 and 1/7 respectively for all the experiments on synthetic images. The conductance function given in Equation 13 is used.

The parameters of FLICM method – local window size, maximum iteration, weighting exponent of membership, threshold of the deviation between cluster centres computed at successive iterations are set as 3, 500, 2, 0.001 respectively.

The misclassification ratio (MCR) is employed to evaluate the performance of the method.

$$MCR = \frac{\text{No. of misclassified pixels}}{\text{Total number of pixels}} \quad (16)$$

In all the three synthetic images shown in **Figure-1 (a), 2 (a) and 3 (a)**, Gaussian noise by 30% is added and shown in **Figure- 1 (b), 2 (b) and 3 (b)**. The outputs of anisotropic filter are shown in **Figure-1 (c), 2 (c) and 3 (c)**. The results of Otsu's threshold segmentation method with anisotropic filtering are shown in **Figure-1 (d), 2 (d) and 3 (d)**. The results of standard EM segmentation without anisotropic filter are shown in **Figure-1 (e), 2 (e) and 3 (e)**. The results of standard EM segmentation with anisotropic filtering are shown in **Figure-1 (f), 2 (f) and 3 (f)**. The results of EM with spatial information segmentation and without anisotropic filtering are shown in **Figure-1 (g), 2 (g) and 3 (g)**. The segmentation results of FLICM are shown in **Figure-1 (h), 2 (h) and 3 (h)** and the segmentation results of the Anisotropic filter based EM with spatial information segmentation are shown in **Figure-1 (i), 2 (i) and 3 (i)**.

The qualitative and quantitative analysis of the results on synthetic images show that integrating anisotropic filtering and EM with local information segmentation provides better segmentation when compared to the methods when applied individually. Especially, the anisotropic filtering contributes more for the segmentation

accuracy while EM with local information provides better tuning in achieving good segmentation accuracy. The method of Otsu's threshold segmentation with anisotropic filtering achieves good results for synthetic image 1 and 2 but it fails to correctly segment synthetic image 3 since the method relies on bimodal histogram and does not consider the probability distribution of the pixels. The FLICM method gives better segmentation accuracy for image with two classes and with less noise. But the segmentation accuracy decreases for image with four classes and with 30% Gaussian noise. The proposed method gives better segmentation accuracy with increasing level of noise and hence the method is robust to noise.

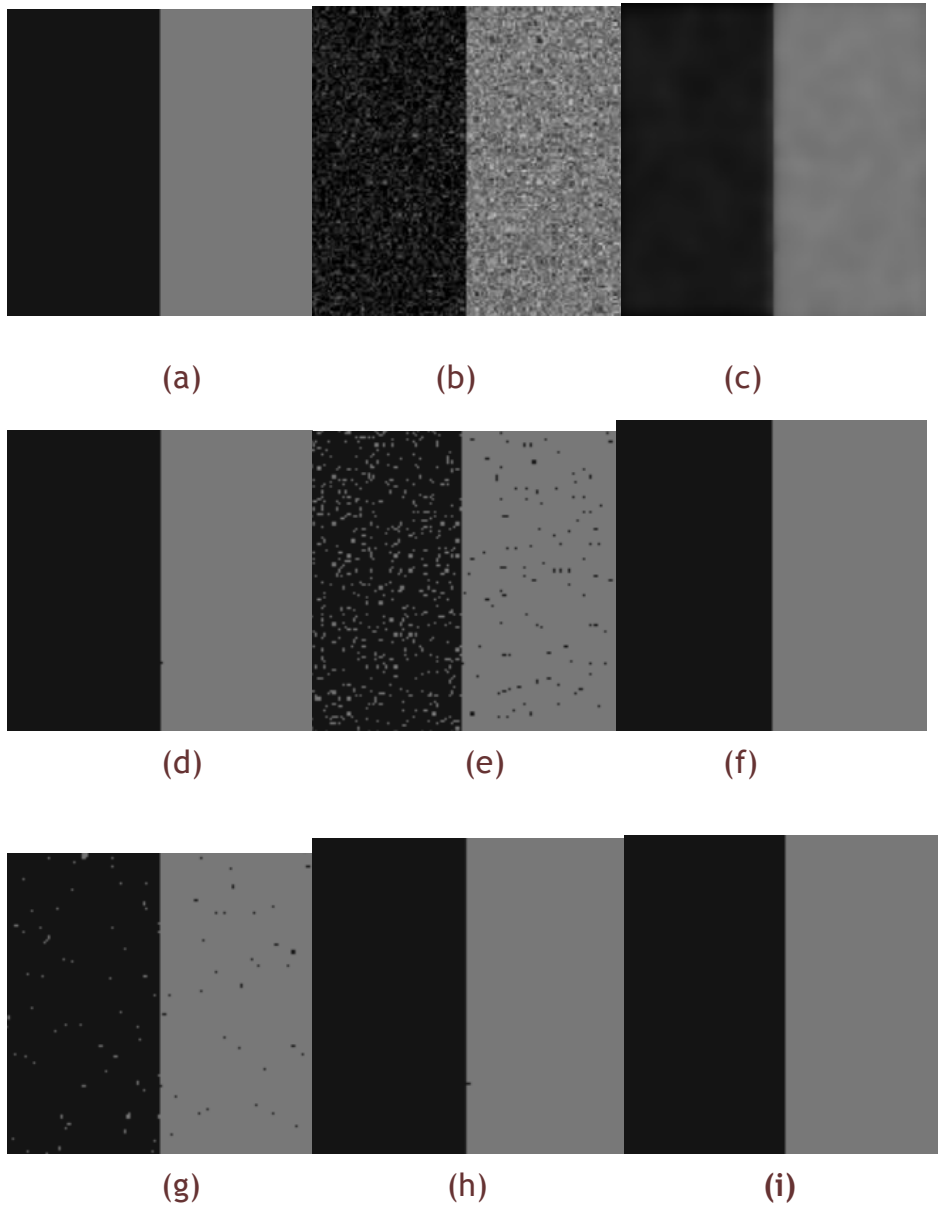


Fig: 1. Segmentation results of the Anisotropic filter based EM with spatial information method on the synthetic Image. (a) Input Image, (b) Image added with Gaussian noise (30%) (PSNR=20.18dB), (c) Anisotropic Filter Output (PSNR=32.79dB), (d) Anisotropic with Otsu's Threshold segmentation (SA=0.99988), (e) Standard EM segmentation without anisotropic filter (SA=0.9552), (f) Standard EM segmentation with anisotropic filter (SA=1.0), (g) EM with spatial information segmentation without anisotropic filter (SA=0.99377), (h) FLICM [9] (SA = 0.99994), (i) Proposed method - Anisotropic filter based EM with spatial information segmentation (SA=1.0)

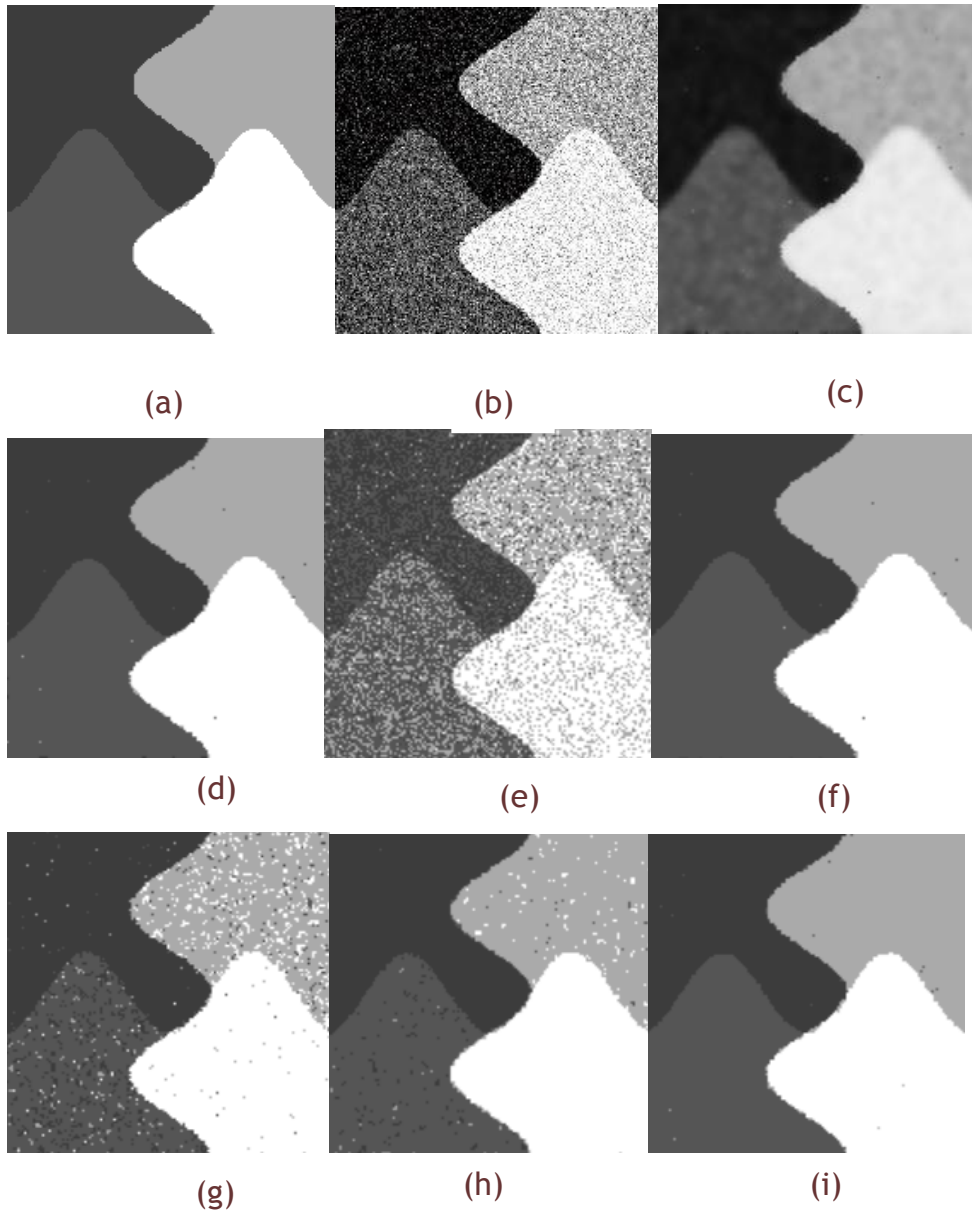


Fig. 2. Segmentation results of the Anisotropic filter based EM with spatial information method on the synthetic Image. (a) Input Image, (b) Image added with Gaussian noise (30%) (PSNR=16.89dB), (c) Anisotropic Filter Output (PSNR=24.6dB), (d) Anisotropic with Otsu's Threshold segmentation (SA=0.9913), (e) Standard EM segmentation without anisotropic filter (SA=0.6446), (f) Standard EM segmentation with anisotropic filter (SA=0.9889), (g) EM with spatial information segmentation without anisotropic filter (SA=0.9144), (h) FLICM [9] (SA = 0.0.9758), (i) Proposed method - Anisotropic filter based EM with spatial information segmentation (0.9912)

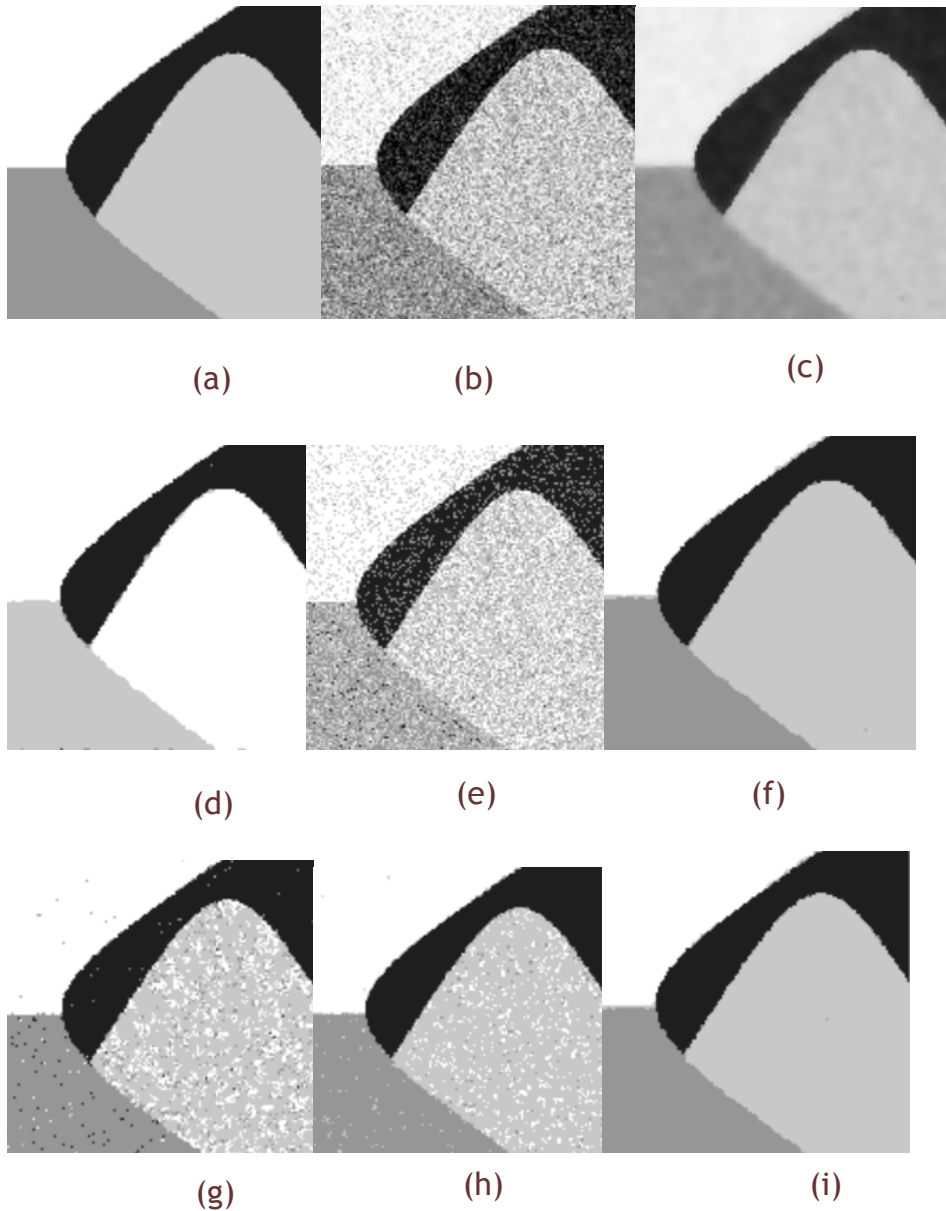


Fig: 3. Segmentation results of the Anisotropic filter based EM with spatial information method on the synthetic Image. (a) Input Image, (b) Image added with Gaussian noise (30%) (PSNR=17.27dB), (c) Anisotropic Filter Output (PSNR=28.13dB), (d) Anisotropic with Otsu's Threshold segmentation (SA=0.3842), (e) Standard EM segmentation without anisotropic filter (SA=0.6484), (f) Standard EM segmentation with anisotropic filter (SA=0.9904), (g) EM with spatial information segmentation without anisotropic filter (SA=0.8787), (h) FLICM [9] (SA =0.9412), (i) Proposed method - Anisotropic filter based EM with spatial information segmentation (0.9922)

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Table: 1. Comparison of Segmentation Accuracy of various methods for Synthetic Image 1

| Methods | Gaussian Noise | | |
|---|----------------|---------|---------|
| | 15% | 20% | 30% |
| Anisotropic with Otsu's Threshold segmentation | 0.99976 | 0.99969 | 0.99988 |
| Standard EM segmentation without anisotropic filter | 0.99396 | 0.98499 | 0.9552 |
| Standard EM segmentation with anisotropic filter | 1.0 | 1.0 | 1.0 |
| EM with spatial information segmentation without anisotropic filter | 0.99902 | 0.99756 | 0.99377 |
| Fuzzy Local Information C Means Clustering (FLICM) | 1.0 | 1.0 | 0.99994 |
| Proposed method – Anisotropic filter based EM with spatial information segmentation | 1.0 | 1.0 | 1.0 |

Table: 2. Comparison of Segmentation Accuracy of various methods for Synthetic Images 2 and 3

| Methods | Synthetic Image 2 | | | Synthetic Image 3 | | |
|---|-------------------|--------|--------|-------------------|--------|--------|
| | Gaussian Noise | | | Gaussian Noise | | |
| | 15% | 20% | 30% | 15% | 20% | 30% |
| Anisotropic with Otsu's Threshold segmentation | 0.9937 | 0.9934 | 0.9913 | 0.3877 | 0.3872 | 0.3842 |
| Standard EM segmentation without anisotropic filter | 0.7346 | 0.6976 | 0.6446 | 0.7608 | 0.7126 | 0.6484 |
| Standard EM segmentation with anisotropic filter | 0.9917 | 0.9907 | 0.9889 | 0.9912 | 0.9915 | 0.9904 |
| EM with spatial information segmentation without anisotropic filter | 0.9628 | 0.9531 | 0.9144 | 0.9612 | 0.9375 | 0.8787 |
| FLICM | 0.9923 | 0.9903 | 0.9758 | 0.9923 | 0.9828 | 0.9412 |
| Proposed method – Anisotropic filter based EM with spatial information segmentation | 0.9938 | 0.9934 | 0.9912 | 0.9926 | 0.9930 | 0.9927 |

Table: 3. Comparison of No. of iterations and execution Time for the methods-Standard EM segmentation with anisotropic filter, RFLCIM and Anisotropic filter based EM with spatial information method and time of execution for anisotropic filter

| Images | Standard EM segmentation with anisotropic filter | | RFLCIM | Anisotropic filter based EM with spatial information Method | | Anisotropic Filter |
|-------------------|--|--------------------------|--------------------------|---|--------------------------|--------------------------|
| | Iteration Count | Time of Execution (secs) | Time of Execution (secs) | Iteration Count | Time of Execution (secs) | Time of Execution (secs) |
| Synthetic Image 1 | 4 | 1.37 | 1.45 | 2 | 1.19 | 0.54 |
| Synthetic Image 2 | 35 | 6.82 | 14.07 | 3 | 3.88 | 1.01 |
| Synthetic Image 3 | 39 | 8.77 | 25.88 | 11 | 5.51 | 1.20 |

From **Table– 1, 2 and 3**, it is inferred that the performance of the Anisotropic filter based EM with spatial information method is superior to the other methods in both execution time and segmentation accuracy.

Experiments on Simulated images from Brainweb

The proposed anisotropic filter based EM with spatial information method is also tested on T1-weighted MR brain images from the BrainWeb database. The method is validated on simulated images with 40% in-homogeneity and 9% noise 181 x 217 x 181 dimension 1 x 1 x 1 mm³ spacing.

The ground truth for the Brain Web dataset is the phantom atlas used to generate the simulated scans. The Dice Similarity Index (DSI) is used as the performance metric. The Dice Similarity Index DSI is given by

$$DSI = \frac{2(P \cap G)}{P + G} \quad (17)$$

Where P represents the sum of pixels classified by the proposed method and G represents the sum of pixels classified by the ground truth and $P \cap G$ represents the sum of pixels classified by both the proposed method and the ground truth.

Two simulated images (#91 and #120) from Brainweb dataset are taken. The brain image is segmented into three classes – Cerebro-spinal Fluid with pixel value 128, Gray matter with pixel value 192 and White matter with pixel value 254. The original images are shown in **Figure–4 (a)** and **Figure– 5 (a)**. The ground truths of the images are shown in **Figure– 4 (b)** and **Figure–5 (b)**. The segmentation results of standard GMM and the anisotropic filter based spatial EM method are shown in **Figure–4 (c), (d) and Figure–5(c), (d)**.

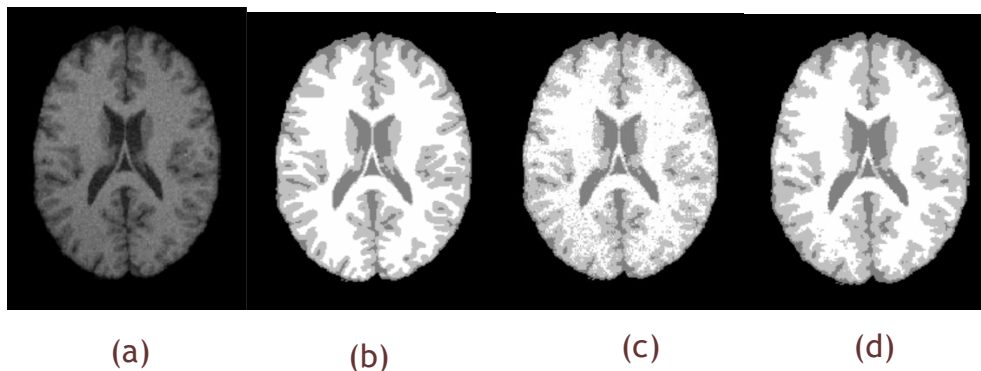


Fig: 4. Segmentation results of the proposed anisotropic filter based EM with spatial information on the MR brain image slice #91. (a) Input Image, (b) Ground truth Image, (c) Standard EM, (d) Anisotropic filter based EM with spatial information method

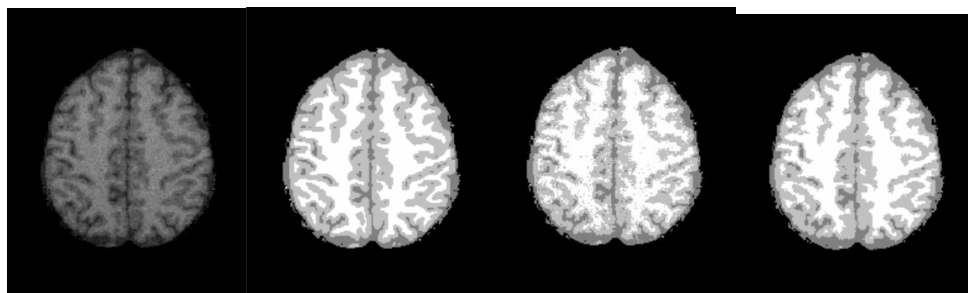


Fig: 5. Segmentation results on the MR brain image slice #120. (a) Input Image, (b) Ground truth Image, (c) Standard EM, (d) Anisotropic filter based EM with spatial information method.

Table 3. Performance Comparison of the Proposed Method to Standard EM Method for Brainweb T1 weighted Images with 40% in-homogeneity and 9% noise slices #91 and #120.

| Class | Dice Similarity Index | |
|-----------|-----------------------|-----------------|
| | Standard GMM | Proposed Method |
| GM (#91) | 0.8305 | 0.8702 |
| WM (#91) | 0.8973 | 0.9339 |
| CSF(#91) | 0.9070 | 0.9067 |
| GM (#120) | 0.8549 | 0.8801 |
| WM (#120) | 0.8756 | 0.9037 |
| CSF(#120) | 0.8965 | 0.9078 |

From **Table-3**, it is inferred the proposed method provides good results for simulated brain images from the Brain web.

CONCLUSION

A new method for segmentation of noisy images based on Gaussian Mixture Model is presented. Anisotropic filter is used for details preserving smoothing. To incorporate the spatial data among the neighboring pixels, the posterior probability is weighted with arithmetic mean filter. The proposed method has been tested on various synthetic and simulated brain images. It is demonstrated that the proposed Anisotropic filter based EM with spatial information method is more efficient both quantitatively and qualitatively compared to the methods in the literature for noisy images. The method is simple and has less computational cost.

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CONFLICT OF INTEREST

Authors declare no conflict of interest.

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