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LEAST SQUARE BASED APPROACH FOR IMAGE INPAINTING

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ABSTRACT

Images are widely used over various applications under the aegis of various domains like Computer vision, Biomedical, etc. The problem of missing data identification is of great concern in various fields involving image processing. Least square can be used for missing sample estimation for 1-D signals. The proposed system extends the missing sample estimation in 1-D using least square to 2-D, applied for image inpainting. The paper also draws a comparison between the Total Variation (TV) algorithm and the proposed method. The experiments were conducted on standard images and the standard metrics namely PSNR and SSIM are used to compare the image quality obtained using the proposed method (least square based) and TV algorithm.

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KEY WORDS

least square; inpainting; missing sample estimation; mask

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INTRODUCTION

Images play the most important role in human perception. In this digital world, we deal with a huge number of images every day. Today, almost all areas of technical endeavor is in some way or the other impacted by digital image processing. An image is a two-dimensional function with two spatial coordinates and a corresponding amplitude level. A digital image is a representation of an image as a set of digital values, called pixels [1].

In real world, the images may be corrupted with letters or scratches. There are also chances for loss of information due to noise or during transmission. In some other cases, we need to remove undesirable objects from the image, say for the case, in which we need to remove an object that destroys the beauty of the image. In order to reconstruct the image free from letters and scratch, or remove an unwanted object, we use image inpainting techniques. Image inpainting is one of the image restoration techniques in which, any lost information is restored using the nearby pixel information. It is considered as the safest way of restoring a degraded image. Digital inpainting has wider applications in image processing, vision analysis and film industry. It can be also be used for old film restoration and red eye correction [2]. The recent applications of image inpainting includes scratch removal from images, text erasing, object removal, disocclusion etc. Generally, the region of the image to be removed, known as mask is defined by the user, while in certain applications, the mask itself need to be detected from the image [3]. Inpainting is known by many names based on their applications, like 'error concealment' in telecommunication [4]. Variational inpainting, texture synthesis, Bayesian inpainting etc. are the most popular inpainting methods. The method is chosen based on the content and location of sampling [5]. In image enhancement problems, the pixel locations contain information regarding the real data as well as noise. Whereas, in case of inpainting, there will be no information related to the real data in the region to be restored [6].

The existing inpainting techniques can be broadly classified into diffusion based approaches and exemplar-based. In diffusion based approaches, linear structures or level lines (isophotes) propagate through diffusion. It is based on the Partial Differential Equations (PDE) and variational methods. The main drawback of the diffusion based method is that, when the region to be filled in is large, the output becomes blurred and is a time consuming process. In exemplar-based method, the best matching texture patches from the surrounding pixels is copied [7]. It works well for larger region restoration. It preserves both the structure and texture of the image [8]. Another inpainting technique that solely relies on PDE is an iterative algorithm. In this method, the information propagates in the direction of minimal change using level sets. Like the diffusion based algorithm, this also doesn't work well for

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large missed regions and is highly time consuming. However, the efficient Total Variation (TV) inpainting method was inspired by this method, which uses anisotropic diffusion and Euler-Lagrange equation, and is based on the strength of isophotes [2]. It was first proposed by Fatemi, Rudin and Osher for image denoising [9]. In TV method, the sharp edges are recovered under some conditions [7]. The computationally expensive and time intensive nature of TV inpainting inhibits its wide spread usage in practical applications [10] [11].

In this paper, the 1-D least square missing sample estimation algorithm is mapped to 2-D images for image inpainting. The image to be inpainted and the mask to be removed are defined by the user. The proposed method uses least square approach for inpainting which not only produce a desired result, but also takes lesser time when compared to the conventional TV algorithm.

Section II is divided into three sub sections, in which the least square method, the mathematical background for the proposed method and missing sample estimation algorithm are discussed. Section III comprises the results which includes the calculated metrics and image outputs. Section IV discusses the importance of proposed method by drawing up an inference out of the results obtained. Section V concludes this paper.

MATERIALS AND METHODS

Least Square Method

The least square method is used to find the best fitting curve solution for a given set of points, mathematically. The basic least square problem is to find the best fitting straight line

$y = Ax + b$, where $x, y \in R^n$;

A is an nxn matrix; and

x, y and b are nx1 vectors

It requires y to be equal to the sum of the linear combination of columns of A and an error vector b. The best fitting curve is estimated by minimizing the sum of squares of the offsets of points from the curve. The sum of squares are used instead of the absolute value so as to take the residuals as a continuous differentiable quantity.

Missing sample estimation

In some cases, the original signal may have missing parts or may be corrupted to the extent where the signal at hand may not even remotely resemble the original signal. This can be due to noise, interference or transmission error. In such problems, the missing samples are to be estimated from the available samples, irrespective of whether the missing sample are random or not. The method can be used independent of whether the missing samples follows a particular structure pattern.

Formulating the problem as a least square problem: [12]

Consider a signal x of length N. Let y be the signal with K number of known samples, where K< N.

$y = Lx$, where L is a KxN selection matrix or sampling matrix

For example, consider a signal of length 5. If only the second, third and last location information is known, then the matrix L can be written as

$$L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The problem stated is, given the signal y and the matrix L, find x such that $y = Lx$. Also, the sum of squares of the second derivative of the whole signal should be minimum.

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$$Lx=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}x(0)\\x(1)\\x(2)\\x(3)\\x(4)\end{bmatrix}=\begin{bmatrix}x(1)\\x(2)\\x(4)\end{bmatrix}=y$$

Vector y has the known samples of x.

$L^T y$ sets the missing samples of x to zero.

$$L^T y=\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\\0&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}y(0)\\y(1)\\y(2)\end{bmatrix}=\begin{bmatrix}0\\y(0)\\y(1)\\0\\y(2)\end{bmatrix}$$

L_c, which is the complement of L is given by taking those rows in the identity matrix that does not appear in L.

$$L_c=\begin{bmatrix}1&0&0&0&0\\0&0&0&1&0\end{bmatrix}$$

An estimate vector $\hat{x}$ can be represented as

$$\hat{x}=L^T y+L_c^T v \tag{1}$$

where v contains the values of the missing samples.
For example,

$$L^T y+L_c^T v=\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\\0&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}y(0)\\y(1)\\y(2)\end{bmatrix}+\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}\begin{bmatrix}v(0)\\v(1)\end{bmatrix}=\begin{bmatrix}v(0)\\y(0)\\y(1)\\v(1)\\y(2)\end{bmatrix}$$

where v is of length N-K and we have to estimate v.

By minimizing $\|F\hat{x}\|_2^2$ we can obtain v, where F is the second order difference matrix given by

$$F=\begin{bmatrix}1&-2&1&&&\\&1&-2&1&&\\&&.&.&&\\&&&.&.&\\&&1&-2&1&\end{bmatrix}$$

Using (1), we can find v by solving the problem

$$\min_v \left\|\mathrm{F}(L^T y+L_c^T v)\right\|_2^2$$

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i.e. $\min\limits_{v} \left\| FL^T y + FL_c^T v \right\|_2^2$

We have, $\min\limits_{q} \left\| p - Hq \right\|_2^2 \;\Rightarrow\; q = (H^T H)^{-1} H^T p$

Let $p = FL^T y$, $H = -FL_c^T$ and $q = v$, then the solution is obtained as

$$v = -(L_c F^T F L_c^T)^{-1} L_c F^T F L^T y \tag{2}$$

After getting v, the estimate $\hat{x}$ can be constructed by using the equation (1).

Proposed Method

The proposed method uses the least square missing sample estimation technique for inpainting. In the proposed system, the missing data estimation which is applicable to 1-D signal is being extended to 2-D images. In the image to be inpainted I (mxn) the pixels corresponding to zero value is replaced by Not a Number (NaN). These are the regions to be inpainted by neighborhood interpolation and are replaced using missing sample estimation using least square algorithm. The missing data estimation method is mapped on to 2-D by taking the image row vectors I (i , :) which is of size 1xn and then implementing the 1-D algorithm is applied to select rows. Similarly, the algorithm is repeated for select column vectors I (: , j)^T of size 1xm of the image matrix. Estimate the number of missing samples. Formulate the selection matrix, a KxN matrix where K is the number of available sample. The selection matrix is basically an identity matrix devoid of the rows corresponding to the missing samples in the signal x. L_c consists of rows of identity matrix not in L. Using equation (2) we obtain the result for both row wise and column wise iterations. The average of the results give the required inpainted image as the result. The flow chart for the algorithm is given in **[Figure-1]**.

Algorithm:

1. Read the image and mask to the variables I (mxn) and M respectively.
2. Resize the mask to the size of the image M (mxn) and convert the mask to gray scale.
3. Convert the mask to binary image.
4. If the mask is white letters in black background, obtain the negative of the image.
   M = 1-M
5. If I is a color image, extract each plane and embed it with mask to create the image to be inpainted.
6. Else if it is a gray image embed the mask directly.
7. Replace all zeros with NaN which are the locations to be inpainted.
8. Do the row wise least square missing sample estimation of the image.
9. If at least one pixel value in that row is NaN, then consider that row.
10. Obtain the number of missing samples, k.
11. Create the sampling matrix, L by choosing the locations where the data is available and take its complement, L_c.
12. Obtain the solution 'x1' using least square algorithm.
    $$x1 = -(L_c F^T F L_c^T)^{-1} L_c F^T F L^T y$$
13. Similarly, do column wise least square missing sample estimation to obtain 'x2'.
14. Take the average of the outputs of step 12 and13 to get the inpainted image.

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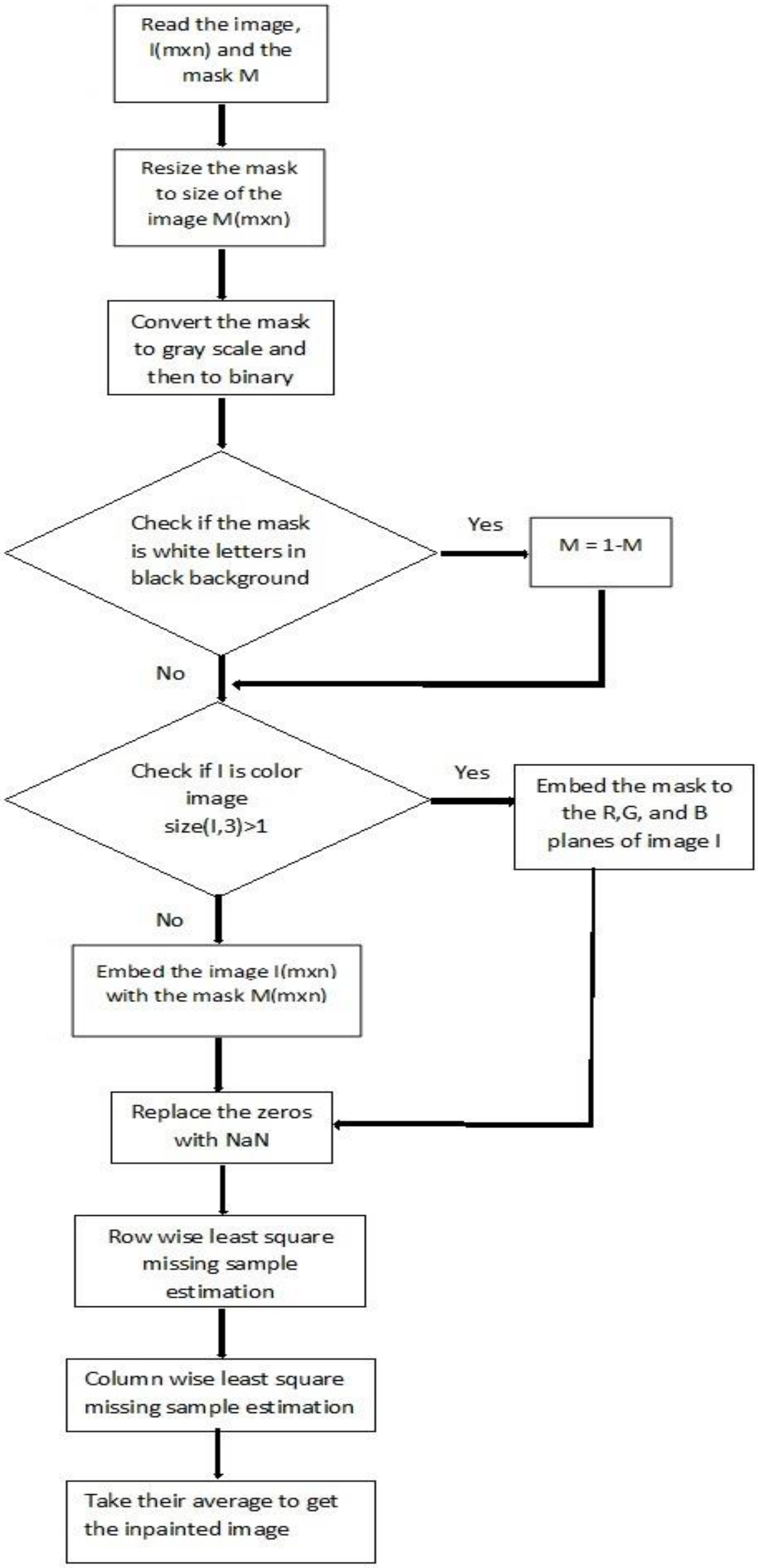

Fig: 1. Flowchart for Algorithm

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DATA SET

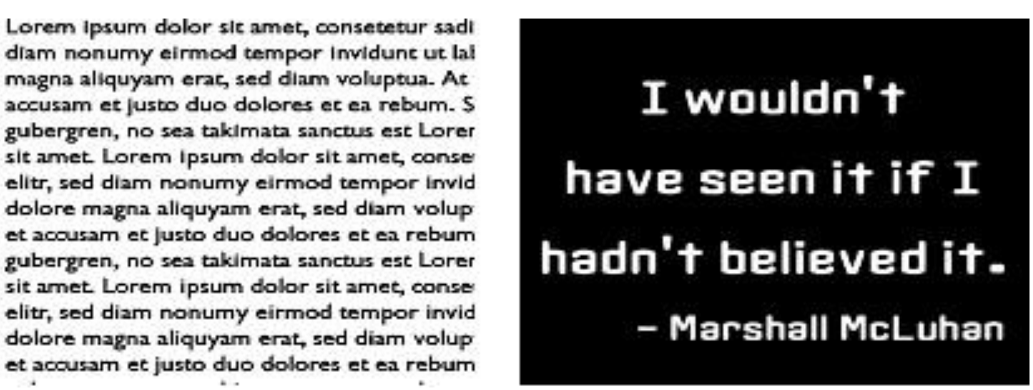
The test images were taken from [13]. The dataset used for the experiment consists of four color images [Figure -2(i)] and five gray scale images [Figure-2(ii)] with letter mask. Two kinds of letter masks, one with black letters on white background and the other with white letters on black background are used [Figure-2(iii)]. We also experimented on another seven images with scratches on them [Figure -2(iv)].

(a) (b) (c) (d)

Fig: 2(i). Input Images Color: (a) Baboon, (b) Lena, (c) Pepper, (d) House

(a) (b) (c) (d) (e)

Fig: 2 (ii). Gray Scale Images: (a) Pirate, (b) Golden bridge, (c) Barbara, (d) Zelda Gifs, (e) Bob cat



(a) (b)

Fig: 2 (iii). Masks: (a) black letters with white background and (b) white letters in black background

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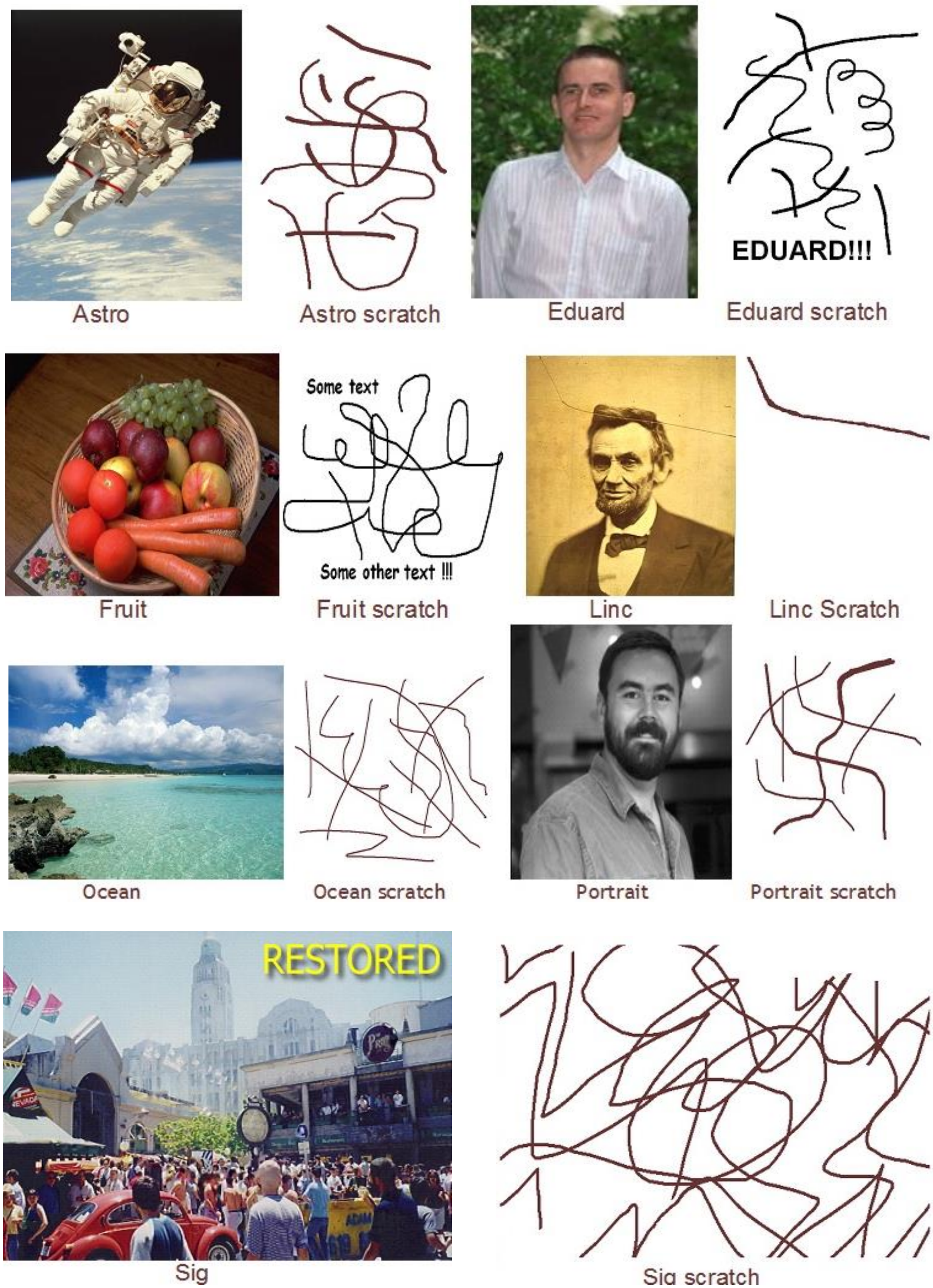

Fig: 2 (iv). Original Images and Masks (Scratch)

RESULTS

The following are the results of the experiments conducted. [Fig- 3(i)] shows the results obtained on color images using the proposed least square based approach. [Fig- 3(ii)] are the outputs obtained for inpainting for gray scale images. In both the cases the images are embedded with text masks (black letters in white background). [Fig -3(iii)] shows the result of the inpainting for images with scratch. Later the algorithm was compared with TV algorithm. The result obtained for inpainting of gray scale image (Bobcat) embedded with white letter mask, using proposed approach and TV algorithm are given in [Fig- 3(v)] and [Fig- 3(iv)] respectively. For the performance evaluation the standard quality matrices PSNR and SSIM are taken. [Table-1] compares the metrics calculated for the color and gray images with letter mask using the proposed least square method. [Table-2] compares the metrics evaluated for the set of seven images for scratch mask and [Table-3] shows the comparison between the proposed least square method and the classical TV method, using the calculated metrics and computation time.

Inpainting using proposed least square approach

Inpainting for color images

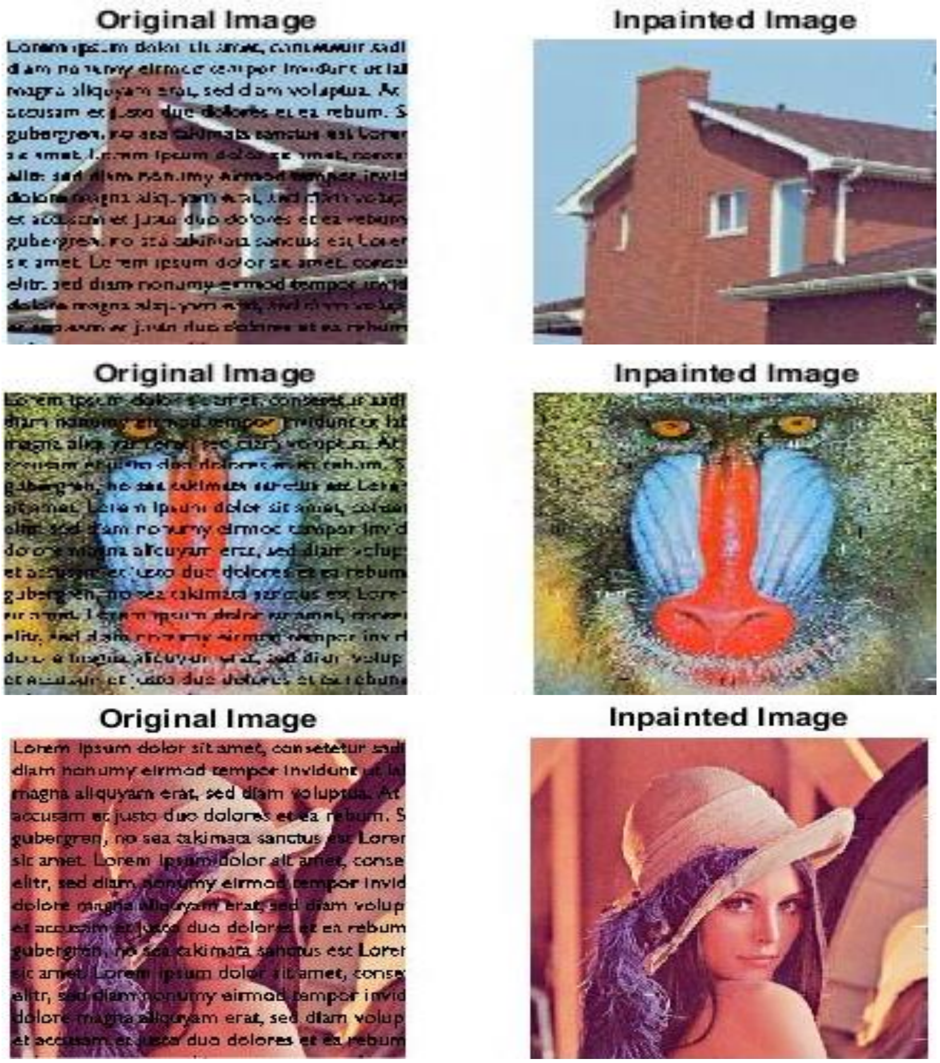

Fig: 3(i). Output of proposed method - Left: Color images with letters to be inpainted; **Right:** Inpainted output

Inpainting for gray scale images

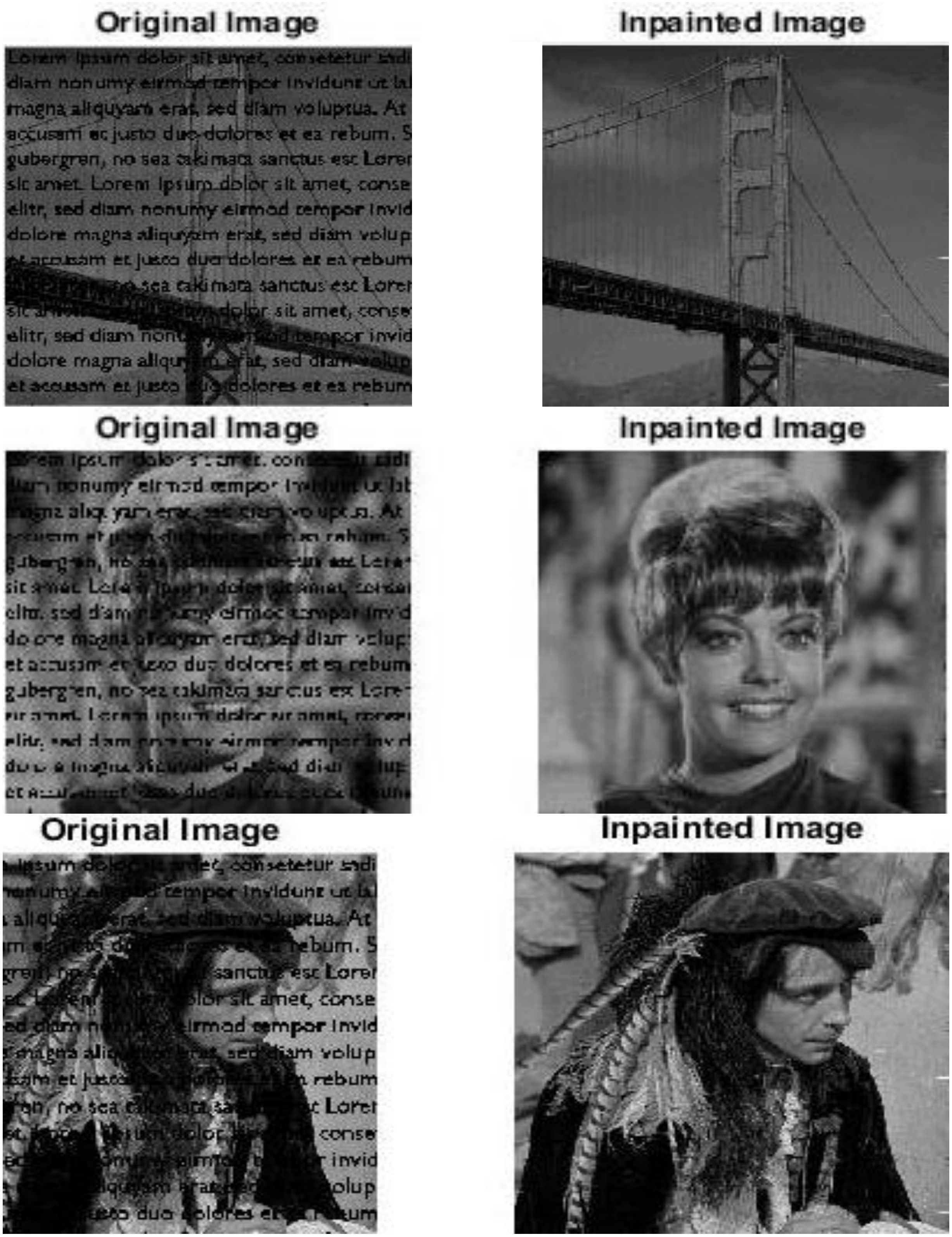

Fig: 3(ii). Output of proposed method - Left: Gray scale images with letters to be inpainted; **Right:** Inpainted output

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Inpainting for color images with scratch

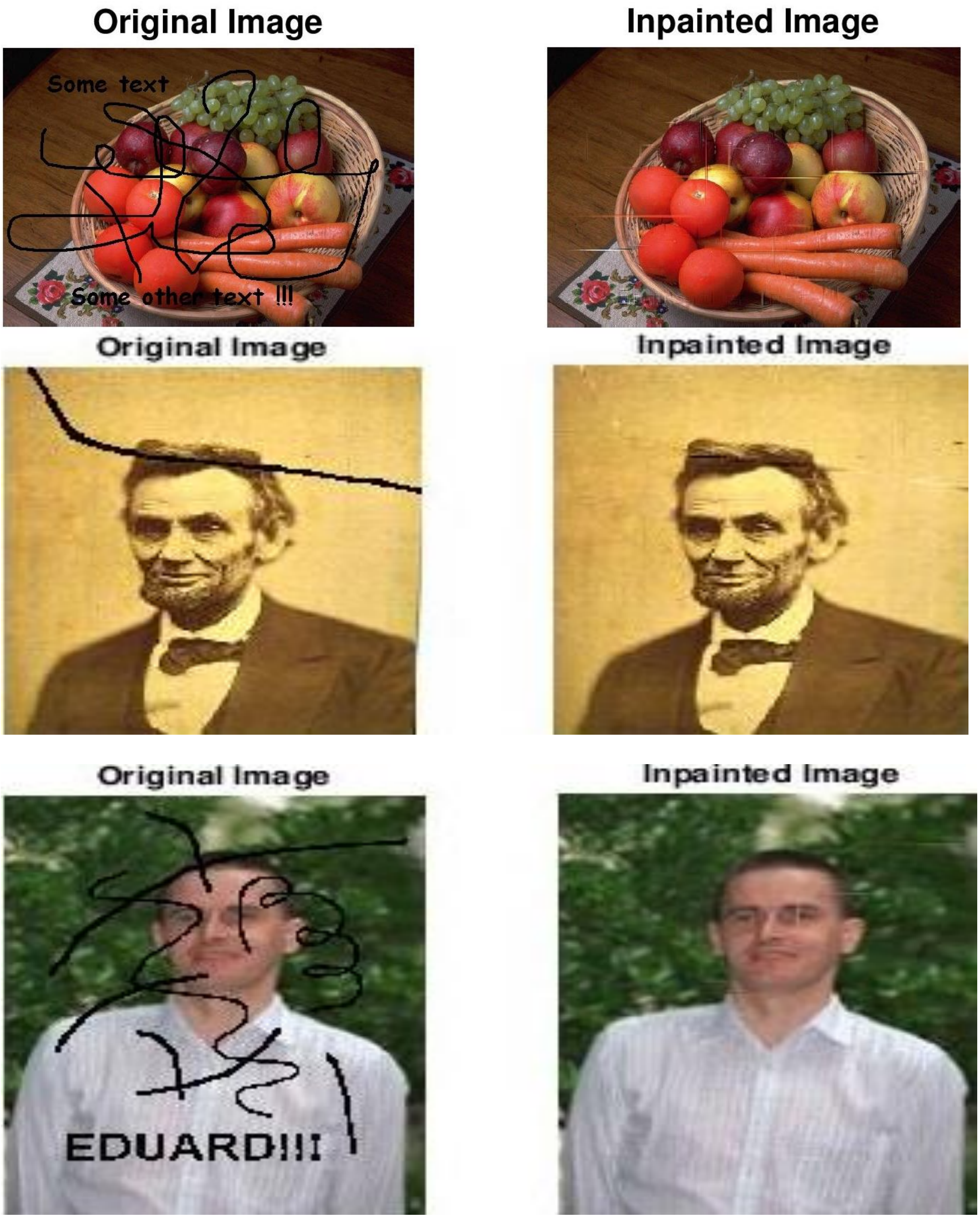

Fig: 3(iii). Output of proposed method - Left: Images with scratch to be inpainted; **Right:** Inpainted output

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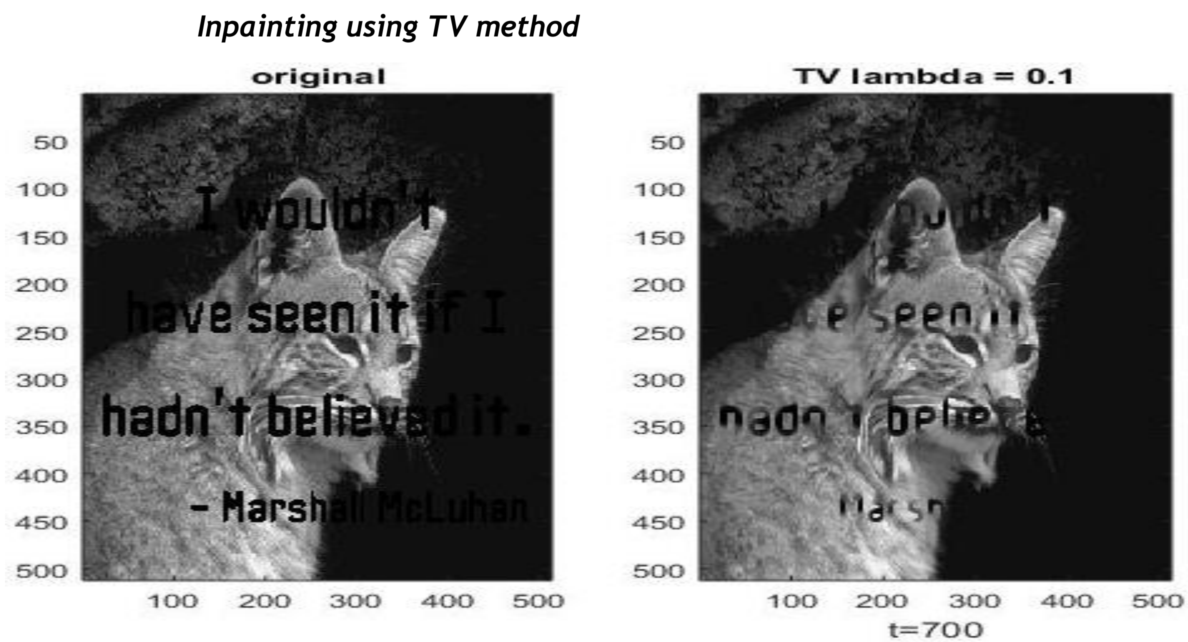

Fig: 3(iv): Output of TV method - Left: Images with letters to be inpainted; **Right:** Inpainted output

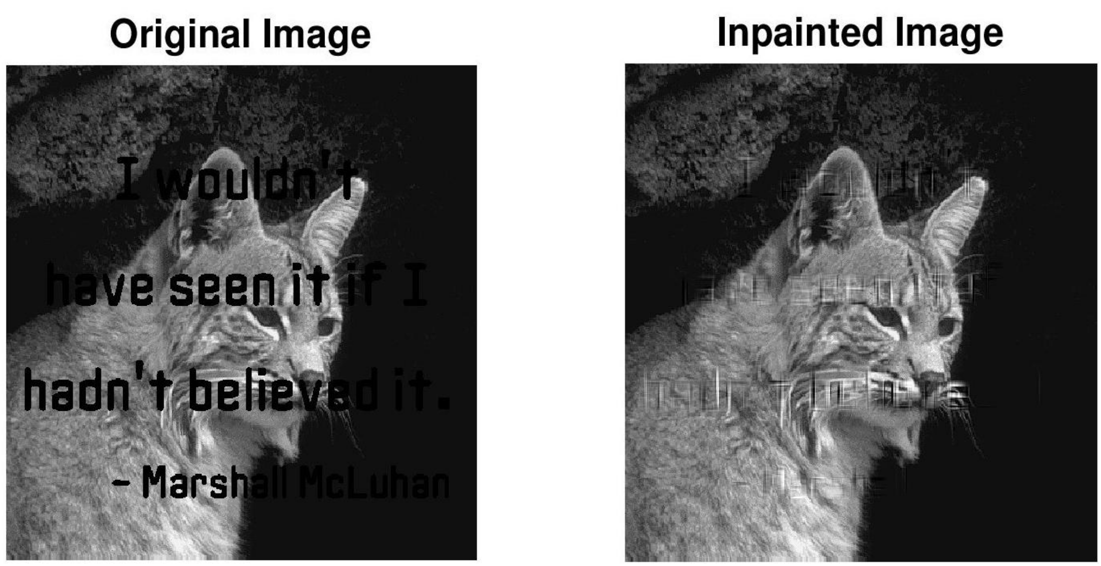

Fig: 3(v). Output of proposed method - Left: Images with white letters to be inpainted; **Right:** Inpainted output

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Table: 1. Computation of PSNR and SSIM for the inpainted color and gray scale images (with letters as mask) obtained using the proposed method

GRAY SCALE IMAGES					COLOUR IMAGES				
IMAGES	PSNR (dB)		SSIM		IMAGES	PSNR (dB)		SSIM	
	ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)	ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)		ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)	ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)
Golden Gate Bridge	7.3575	23.2524	0.4849	0.8327	House	7.1201	27.7246	0.4923	0.9805
Zelda Gifs	7.2768	28.0292	0.4953	0.8864	Baboon	7.3047	18.0012	0.6621	0.9004
Barbara	7.2491	19.7616	0.5364	0.8513	Lena	7.3373	26.2778	0.6725	0.9859
Bobcat (Black text mask)	7.3458	21.9375	0.5103	0.8678	Pepper	7.3232	18.0611	0.7094	0.9568
Bobcat (White text mask)	9.4269	18.8585	0.8286	0.9204					
Pirate	7.4524	19.7506	0.6228	0.8283					

Table: 2. Computation of PSNR and SSIM for the inpainted color and gray scale images (with scratch as mask) obtained using the proposed method

SCRATCH				
IMAGES	PSNR (dB)		SSIM	
	ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)	ORIGINAL	INPAINTED IMAGE (PROPOSED METHOD)
Astro	10.3396	16.7556	0.8539	0.9441
Eduard	10.7427	36.4272	0.8501	0.9879
Fruit	10.1785	19.0196	0.8921	0.9837
Linc	18.0899	29.4684	0.9781	0.9952
Ocean	12.5589	21.8784	0.8936	0.9806
Portrait	12.2981	22.4735	0.8663	0.943
Sig	9.2693	19.2938	0.7791	0.9188

DISCUSSION

The main objective of this paper is to show that the least square missing sample estimation method mapped for inpainting gives better result in no time when compared to the traditional total variation method. [Fig-4] shows that the elapsed time for the proposed method is much less than that for TV method. The TV method was experimented only on the gray scale images, which itself took a considerable amount of time to obtain the result. The maximum time taken for inpainting on gray scale images with letter mask, using the proposed method was 4.392885s whereas, the minimum time for the TV method was 380.051024s. Apart from having less computational time, the proposed

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model gives an output of good visual quality when compared to the outputs obtained for the TV inpainting as shown in **[Figure -3(v)]**. The output of the TV inpainting is blurred. Also from **[Figure-5]** and **[Figure -6]**, we can infer that the PSNR and SSIM values are impressible for the proposed method. The PSNR computed for the 'Bob cat' image using the TV and proposed method are 20.7637dB and 21.9375dB respectively. Also SSIM for the image using the two methods are 0.4715 (TV) and 0.8678 (proposed method) **[Table-3]**. We can see that there is an improvement of 1.1738dB in PSNR and 0.3963 in SSIM. This improvement can be verified visually by the output images shown in **[Figure-3(iv)]** and **[Figure-3(v)]**. From this comparison, we can conclude that the proposed least square based approach is more appropriate for image inpainting

Table: 3. Performance comparison of the proposed technique for image inpainting against the TV algorithm based on PSNR, SSIM and the computational time

GRAY SCALE IMAGES						
IMAGES	PSNR (dB)		SSIM		TIME (s)	
	T V	PROPOSED METHOD	TV	PROPOSED METHOD	TV	PROPOSED METHOD
Golden gate bridge	20.718	23.2524	0.3863	0.8327	392.083227	1.316435
Zelda gifs	21.6133	28.0292	0.7602	0.8864	406.260955	1.020285
Barbara	17.1117	19.7616	0.3508	0.8513	420.562651	4.392885
Bobcat	20.7637	21.9375	0.4715	0.8678	380.051024	4.18181
Pirate	16.5703	19.7506	0.2519	0.8283	397.130245	1.408976

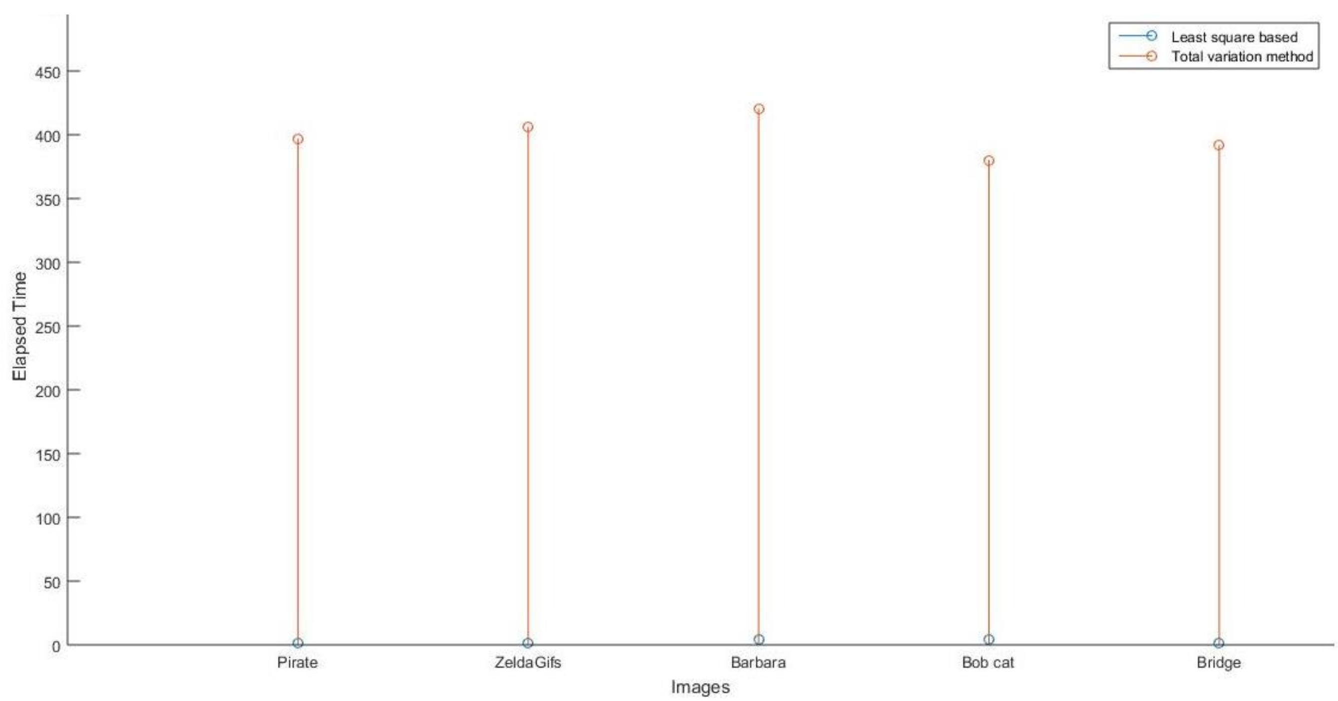

Fig: 4: Comparison of the elapsed time for TV method and the proposed method

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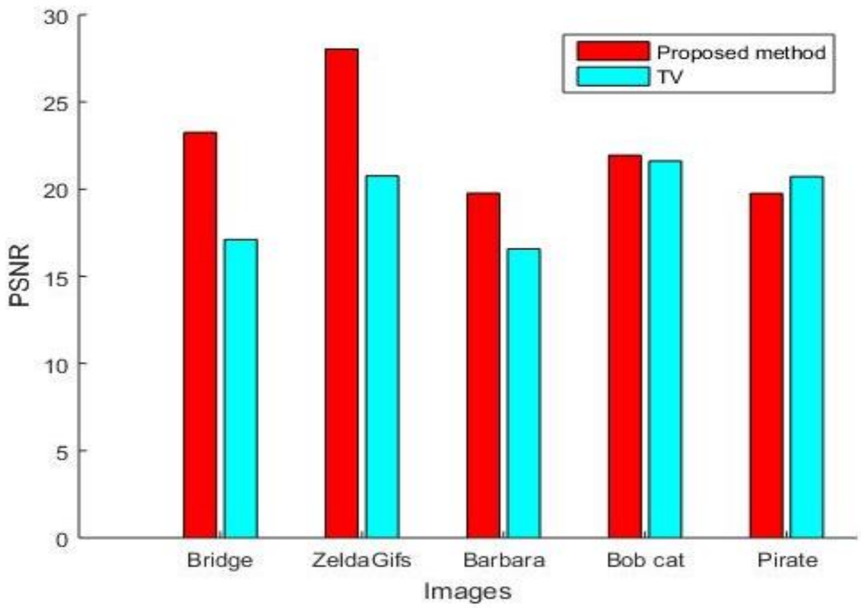

Fig: 5. Comparison of the PSNR values for the proposed method and TV based on **[Table-3]**

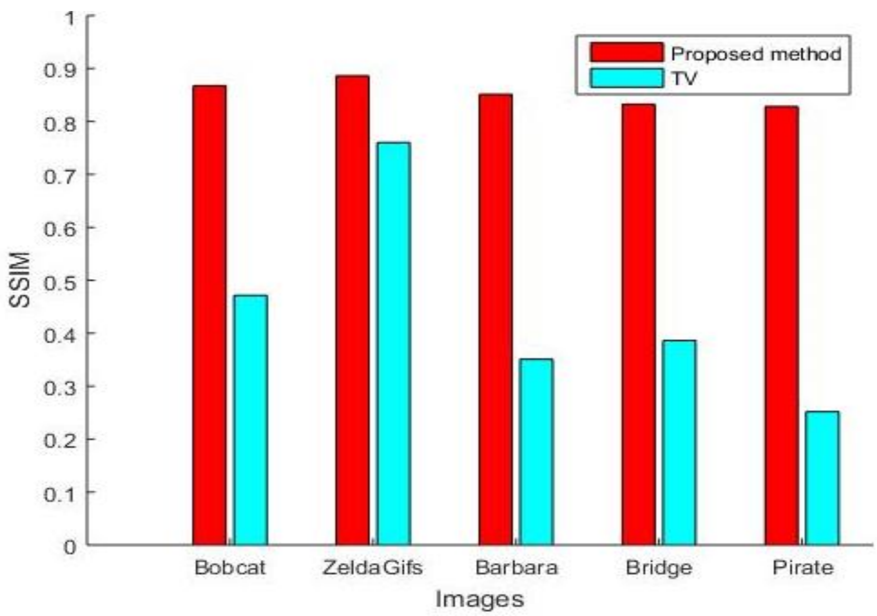

Fig: 6. Comparison of the SSIM values for the proposed method and TV based on **[Table-3]**

The comparison between the PSNR values of the original image and the inpainted image is shown in [Fig-7]. It can be seen from the graph that there is a considerable increase in the PSNR value. The SSIM value comparison is shown in [Fig-8]. It can be seen that the SSIM values of all the inpainted images are nearly equal to 1. Using the proposed approach the PSNR and SSIM has been improved, on an average to 13.429dB and 0.214 respectively.

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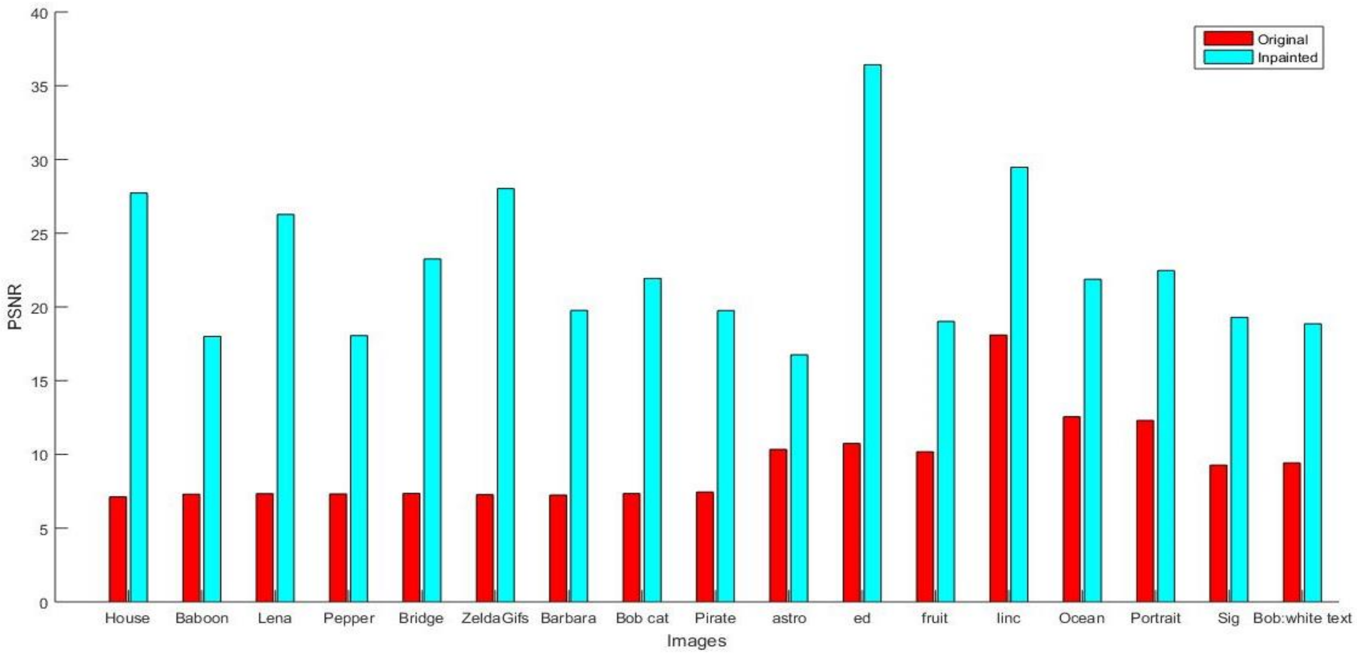

Fig: 7. Comparison the PSNR values of the original and the inpainted image based on **[Table-1]** and **[Table-2]**

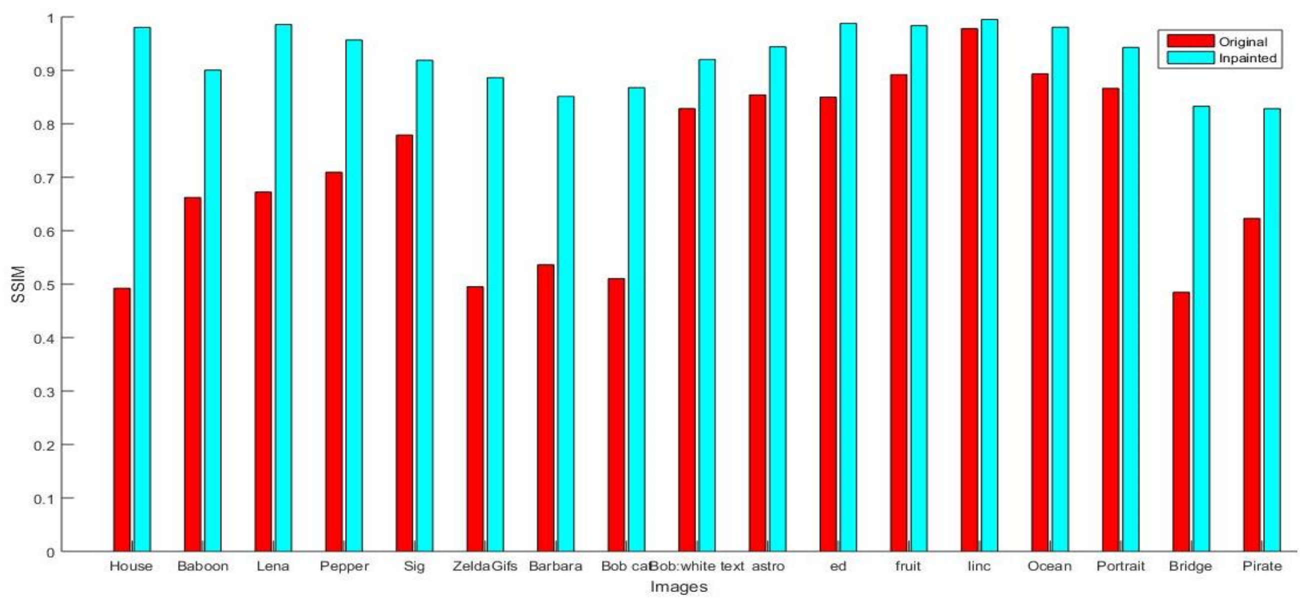

Fig: 8. Comparison of the SSIM values of the original and the inpainted image based on [Table-1] and [Table-2]

CONCLUSION

This paper presents a least square based approach for image inpainting. The least square based missing sample estimation method for 1-D signal is being extended to 2-D images for inpainting. The proposed approach is experimented on standard test images embedded with letter and scratch masks and the results are compared with TV

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method. The efficiency of the proposed approach is assessed by computing standard quality metrics such as PSNR and SSIM. It shows that our approach performs well for images covered with letter and scratch masks. Also the time complexity of our approach is much less compared to the classical TV method. The drawback of the proposed approach is that, it fails for patch based inpainting. In future direction, we can extend this method for patch based image inpainting.

CONFLICT OF INTEREST

Authors declare no conflict of interest.

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None.

FINANCIAL DISCLOSURE

No financial support was received to carry out this project.

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