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APPROXIMATE EQUALITIES FOR COVERING BASED OPTIMISTIC MULTI GRANULAR ROUGH SETS AND THEIR PROPERTIES

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ABSTRACT

Imprecision in modern day data has become a common feature and in order to efficiently handle them many uncertainty based models have been put forth in the literature. Rough set model introduced by Pawlak has established itself to be an efficient model in many real life situations. But the basic rough set model of Pawlak has limited applications because of the constraint of it being dependent on equivalence relations. The equivalent mathematical concept to equivalence relation is that of a partition. A cover is a generalization of the notion of partition and this led to the development of covering based rough sets, which has better modeling power than basic rough sets. Following the concepts of granular computing rough set introduced by Pawlak is single granulation. So, in order to handle multi-granularity, two types of multi-granularities called optimistic multi-granulation and pessimistic multi-granulation were introduced in 2006 and 2010 respectively. Recently these two concepts of multi-granulation and covering based rough sets were combined to define covering based multi-granular rough sets. The equality of sets in mathematics is too redundant to have any fruitful real life application as it does not include user knowledge into it, which is normally done in practice. In order to handle this rough equalities were defined by Pawlak et al, which was extended by Tripathy in2008 to define rough equivalence. In this paper we introduce and study covering based optimistic multi-granular approximate equalities and study their properties. We study two types of properties called general properties and replacement properties. A real life example is used for illustration of the concepts and also to aid in the construction of counter examples in the proofs of the properties.

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INTRODUCTION

Data in real life are mostly imprecise in nature and so the conventional tools for formal modeling, reasoning and computing, which are crisp, deterministic and precise in characteristics, are inadequate to handle them. This gives rise to the development of several imprecise models, of which rough sets introduced by Pawlak [5, 6] is one of the most efficient one. It has been proved to be very efficient to capture impreciseness in data. According to Pawlak, the knowledge of human beings depends upon their capability to classify objects of universes. Equivalence relations on any universe induce classifications through the equivalence classes associated with them. So, Pawlak had taken equivalence relations to define rough sets and related notions.
A pair of crisp sets called the lower and upper approximations are associated with every rough set. Lower approximation comprising of elements certainly belong to it and upper approximation comprising of elements certainly or possibly belong to it, with respect to the available information.

This basic rough set has been extended further in many directions. These extensions are actually either based on tolerance relations or any such relations that do not require the stringent restrictions of an equivalence relation.

From the point of view of granular computing, basic rough set theory deals with a single granulation [17]. However, in some application areas we need to handle more than one granulation at a time and this necessitated the development of multi-granular rough sets (MGRS)[7], where at least two equivalence relations are taken for granulation of a universe. This concept is further extended by considering covers and this lead to the development of covering based multi granular rough sets(CBMGRS). They are of two types namely, optimistic and pessimistic. In this paper optimistic one is considered. Four types of CBOMGRS are defined and their properties, general and replacement, are established.

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Equality of sets in mathematics is a very stringent notion and its application is also limited. In real life situations we use our own knowledge about the universe of discourse to determine the equality of sets. However, such interception of user in deciding the equality of sets is very much expected. In order to make a place for user knowledge in deciding the equality of sets fully or partially, three kinds of rough equalities were introduced by Novotny and Pawlak [5,6]. In fact three notions were introduced, called the top rough equality, the bottom rough equality and the rough equality. Now the sets can be equal or not from the user point of view. They had established several properties of these notions. They tried to interchange the concepts of top rough equality and bottom rough equality in the properties to find their validity and commented that these properties do not hold true under such circumstances. .

This paper is organized into four sections. First section provides the over view and related literatures. Section two presents various definitions and notions required. Section three introduces multi granular rough equalities and its general and replacement properties. In this section real life examples are considered to prove few properties as sample. In section four conclusion to the work are presented.

DEFINITIONS AND NOTATIONS

Rough set

The notion of rough set was introduced by Z.Pawlak in the year 1982 ([5]). We extract the definition and present below.

Let *U* be a universe of discourse and *R* be an equivalence relation over *U*. By *U/R* we denote the family of all equivalence classes of *R*, referred to as categories or concepts of *R* and the equivalence class of an element $x \in U$ is denoted by $[x]_R$. By a knowledge base, we understand a relational system $K = (U, P)$, where *U* is as above and P is a family of equivalence relations over *U*. For any subset Q ($\neq \phi$) $\subseteq$ P, the intersection of all equivalence relations in Q is denoted by IND(Q) and is called the indiscernibility relation over Q. Given any $A \subseteq U$ and R $\in$ IND (K), we associate two subsets, $\underline{R}A = \bigcup\{B \in U/R : B \subseteq A\}$ and $\overline{R}A = \bigcup\{B \in U/R : B \bigcap A \neq \phi\}$, called the R-lower and R-upper approximations of 'A' respectively. The R-boundary of 'A' is denoted by BN_R *(A)* and is given by $BN_R(A) = \overline{R}A - \underline{R}A$. The elements of $\underline{R}$ A are those elements of U, which can certainly be classified as elements of A, and the elements of $\overline{R}$ A are those elements of U, which can possibly be classified as elements of 'A', employing knowledge of *R*. We say that A is rough with respect to R if and only if $\underline{R}A \neq \overline{R}A$, equivalently $BN_R(A) \neq \phi$. 'A' is said to be R-definable if and only if $\underline{R}A = \overline{R}A$, or $BN_R(A) = \phi$.

Covering based rough sets

Basic rough sets introduced by Pawlak have been extended in many ways. One such extension is the notion of covering based rough sets, where the notion of partitions is replaced by the general notion of covers [16]. In this section we introduce the basics of these sets.

Definition 2.2.1: ([23, 25]) Let U be a universe and C = {C_1,C_2,….., C_n} be a family of non-empty subsets of U that may be overlapping in nature. If $\bigcup C$ = U, then C is called a covering of U. The pair (U, C) is called covering approximation space. For any A $\subseteq$ U, the covering lower and upper approximations of 'A' with respect to C can be defined as follows

$$(2.2.1)\quad \underline{C}(A) = \bigcup\{C_i \subset A, i \in \{1, 2, ..., n\}\}$$

$$(2.2.2)\quad \overline{C}(A) = \bigcup\{C_i \cap A \neq \phi, i \in \{1, 2, ..., n\}\}$$

The pair $(\underline{C}(A), \overline{C}(A))$ is called covering based rough set associated with X with respect to cover C if $\underline{C}(A) \neq \overline{C}(A)$, i.e., A is said to be roughly definable with respect to C. Otherwise A is said to be C-definable.

Definition 2.2.2: ([23,25]) Given a covering approximation space (U, C) for any x $\in$ U, sets $md_c(x)$ and $MD_c(x)$ are respectively called minimal and maximal descriptors of x with respect to C,

$$(2.2.3)\quad md_c(x) = \{S \in \boldsymbol{C} \,/\, x \in S \text{ and } (\forall T \in \boldsymbol{C} \text{ if } (x \in T \text{ and } T \subseteq S) \text{ then } S = T)\}$$

It is a set of all minimal covers containing x where a minimal cover containing x be one for which no proper sub cover containing x exists.

$$(2.2.4)\quad MD_c(x) = \{S \in \boldsymbol{C} \,/\, x \in S \text{ and } (\forall T \in \boldsymbol{C} \text{ if } (x \in T \text{ and } T \supseteq S) \text{ then } S = T)\}$$

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It is a set of all maximal covers containing x where a maximal cover containing x be one for which no proper super cover containing x exists.

Multi granular rough sets

In the view of granular computing (proposed by L. A. Zadeh), an equivalence relation on the universe can be regarded as a granulation, and a partition on the universe can be regarded as a granulation space [5, 6]. For an incomplete information system, similarly, a tolerance relation on the universe can be one regard as a granulation, and a cover induced by the relation can be regarded as a granulation space. Several measures in knowledge base closely associated with granular computing, such as knowledge granulation, granulation measure, information entropy and rough entropy. On research of rough set method based on multi-granulations, Y. H. Qian and J. Y. Liang introduced a rough set model based on multi-granulations [7], which is established by using multi equivalence relations.

Definition 2.3.1 ([23, 25]) Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, T, S $\in$ **R.** We define the optimistic multi-granular lower approximation and upper approximation of X in U as

$$(2.3.1)\quad \underline{T+S}(A)=\bigcup\{x/[x]_T\subseteq A \; or \; [x]_s\subseteq A\}$$ and

$$(2.3.2)\quad \overline{T+S}(A)=(\underline{T+S}(A^C))^C$$

Covering based optimistic multi granular rough sets

The notion of Multi-granular rough sets have been extended to covering approximation spaces. They can be of two types; namely, optimistic and pessimistic. By employing minimal and maximal descriptors four types of CBOMGRS are possible. The definitions of four types of CBOMGRS are as follows [4].

Let (U, C) be a covering approximation space, C_1 and C_2 be two covers in C and A be any subset of U, The four types of optimistic covering based optimistic multi granular rough sets, are defined as follows.

Definition 2.4.1([16]): The first type CBOMGRS lower and upper approximations with respect to C_1 and C_2 are defined as

$$(2.4.1)\quad \underline{O_{C_1+C_2}}(A)=\{x\in U/\bigcap md_{C_1}(x)\subseteq A \;\; or \;\; \bigcap md_{C_2}(x)\subseteq A\}$$ and

$$(2.4.2)\quad \overline{O_{C_1+C_2}}(A)=\{x\in U/(\bigcap md_{C_1}(x))\cap A\neq\phi \;\; and \;\; (\bigcap md_{C_2}(x))\cap A\neq\phi\}$$

Definition 2.4.2([16]): The second type CBOMGRS lower and upper approximations with respect to C_1 and C_2 are defined as

$$(2.4.3)\quad \underline{S_{C_1+C_2}}(A)=\{x\in U/\bigcup md_{C_1}(x)\subseteq A \;\; or \;\; \bigcup md_{C_2}(x)\subseteq A\}$$ and

$$(2.4.4)\quad \overline{S_{C_1+C_2}}(A)=\{x\in U/(\bigcup md_{C_1}(x))\cap A\neq\phi \;\; and \;\; (\bigcup md_{C_2}(x))\cap A\neq\phi\}$$

Definition 2.4.3([16]): The third type CBOMGRS lower and upper approximations with respect to C_1 and C_2 are defined as

$$(2.4.5)\quad \underline{T_{C_1+C_2}}(A)=\{x\in U/\bigcap MD_{C_1}(x)\subseteq A \;\; or \;\; \bigcap MD_{C_2}(x)\subseteq A\}$$ and

$$(2.4.6)\quad \overline{T_{C_1+C_2}}(A)=\{x\in U/(\bigcap MD_{C_1}(x))\cap A\neq\phi \;\; and \;\; (\bigcap MD_{C_2}(x))\cap A\neq\phi\}$$

Definition 2.4.4([16]): The fourth type CBOMGRS lower and upper approximations with respect to C_1 and C_2 are defined as

$$(2.4.7)\quad \underline{L_{C_1+C_2}}(A)=\{x\in U/\bigcup MD_{C_1}(x)\subseteq A \;\; or \;\; \bigcup MD_{C_2}(x)\subseteq A\}$$ and

$$(2.4.8)\quad \overline{L_{C_1+C_2}}(A)=\{x\in U/(\bigcup MD_{C_1}(x))\cap A\neq\phi \;\; and \;\; (\bigcup MD_{C_2}(x))\cap A\neq\phi\}$$

Properties of covering based optimistic multi granular rough sets

The following are the properties of covering based optimistic multi granular rough sets. Here 'w' denotes any of the four types first, second, third or fourth of optimistic multigranulation. Let A and B be any two subsets of U. We omit the proofs of these properties as these are more or less trivial. The proofs can also be found in [15,16].

$$(2.5.1)\qquad A\subseteq B \Rightarrow \underline{W_{C_1+C_2}}(A)\subseteq \underline{W_{C_1+C_2}}(B)$$

$$(2.5.2)\qquad A\subseteq B \Rightarrow \overline{W_{C_1+C_2}}(A)\subseteq \overline{W_{C_1+C_2}}(B)$$

$$(2.5.3)\qquad \underline{W_{C_1+C_2}}(\sim A)=\sim \overline{W_{C_1+C_2}}(A)$$

$$(2.5.4)\qquad \overline{W_{C_1+C_2}}(\sim A)=\sim \underline{W_{C_1+C_2}}(A)$$

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(2.5.5) $\underline{W_{C_1+C_2}}(A \cup B) \supseteq \underline{W_{C_1+C_2}}(A) \cup \underline{W_{C_1+C_2}}(B)$

(2.5.6) $\overline{W_{C_1+C_2}}(A \cup B) \supseteq \overline{W_{C_1+C_2}}(A) \cup \overline{W_{C_1+C_2}}(B)$

(2.5.7) $\underline{W_{C_1+C_2}}(A \cap B) \subseteq \underline{W_{C_1+C_2}}(A) \cap \underline{W_{C_1+C_2}}(B)$

(2.5.8) $\overline{W_{C_1+C_2}}(A \cap B) \subseteq \overline{W_{C_1+C_2}}(A) \cap \overline{W_{C_1+C_2}}(B)$

RESULTS

Approximate equalities

The equality of sets or domains used in mathematics is too stringent. In most of the real life situations we often consider equality of sets or domains, as approximately equal under the existing circumstances. These existing circumstances serve as user knowledge about the set or domain. So, they play a significant role in approximate reasoning. Also, one can state that it mostly depends upon the knowledge the assessors have about the set of domains under consideration as a whole but not on the knowledge about individuals of the sets or domains.

As a step to include user knowledge in considering likely equality of sets, Novotny and Pawlak [5,6] introduced the following rough equalities of two sets A and B which are subsets of U.

Let K= (U, R) be a knowledge base, $A, B \subseteq U$ *and* $R \in IND(K)$.

Definition 3.1: We say that,

(3.1.1) A and B are bottom rough equal (A $\overline{\overline{R}}$ B) if and if only $\underline{R}A = \underline{R}B$.

(3.1.2)A and B are top rough equal (A $\underline{\underline{R}}$ B) if and if only $\overline{R}A = \overline{R}B$.

(3.1.3)A and B are rough equal (A R_eq B) if and if only $\underline{R}A = \underline{R}B$ and $\overline{R}A = \overline{R}B$ i.e., (A $\overline{\overline{R}}$ B) and (A $\underline{\underline{R}}$ B).

There are several properties of these approximate equalities established by Novotny and Pawlak in the form of general and replacement properties. The replacement properties are those properties obtained from the general properties by interchanging the top and bottom equalities. As noted by them, all these approximate equalities of sets are relative in character; that is, sets are equal or not equal from our point of view depending on what we know about them. So, in a sense the definition of rough equality incorporates user knowledge about the universe in arriving at equality of sets or domains. However, these notions of approximate equalities of sets boil down to equality of sets again. So, in order to make the equalities more general, a notion called rough equivalences was introduced by Tripathy in 2008 [16]. These notions are more general and more applicable in real life situations. An example of cattle in a society is taken by him to explain the drawbacks in the earlier notion and also to establish the superiority of the new notions in the real life scenario. These two different forms of approximate equalities have been generalized to the context of multi-granulation by Tripathy along with coauthors in a series of papers [14-17, 24-26].

In this paper we shall introduce the concepts of approximate equalities and rough equivalence to the context of covering based optimistic multi granulations and prove their properties (both general and replacement). We establish both the direct as well as the replacement properties for both these notions. In fact two types of covering based multi granular rough equalities and equivalences are possible, namely, optimistic and pessimistic. In this paper optimistic ones are considered. First type covering based optimistic multi granular rough set is considered and its rough equalities and equivalences are studied. The direct properties of such sets are stated and proved first. Later its replacement properties are also studied and proved. To substantiate better understanding of these concepts few of these properties are studied and interpreted in terms of one real life example.

Covering based optimistic multi granular approximate equalities

We introduce in the following the different covering based optimistic multi granular rough equalities for first type, CBOMGRS, and study their properties. For the other types of multi granulations similar definitions hold good.

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Let C_1 and C_2 be two covers on U and let O denotes first type of CBOMGRS.

Definition 3.2: We say that,

(3.2.1) A and B are bottom C_1+C_2 rough equal to each other (A $\underset{C_1+C_2}{\overset{=}{}}$ B) iff $\underline{O_{C_1+C_2}}(A) = \underline{O_{C_1+C_2}}(B)$.

(3.2.2) A and B are top C_1+C_2 rough equal to each other (A $\overset{C_1+C_2}{\underset{=}{}}$ B) iff $\overrightarrow{O_{C_1+C_2}}(A) = \overrightarrow{O_{C_1+C_2}}(B)$.

(3.2.3) A and B are optimistic total rough equal to each other with respect to C_1 and C_2 (A *r_C_1+C_2_eq* B) iff $\underline{O_{C_1+C_2}}(A) = \underline{O_{C_1+C_2}}(B)$ and $\overrightarrow{O_{C_1+C_2}}(A) = \overrightarrow{O_{C_1+C_2}}(B)$.

Properties for first type of covering based optimistic multi granular approximate equalities

The general properties of first type of covering based rough equalities are stated, proved and substantiated with few proofs and examples wherever necessary.

Let C_1 and C_2 be two covers on U and $C_1, C_2 \in C$ and $A, B \subseteq U$. Let F denotes first type CBOMGRS. Then

(3.3.1) A $\underset{C_1+C_2}{\overset{=}{}}$ B if $A \cap B$ $\underset{C_1+C_2}{\overset{=}{}}$ A and B both. But the converse may not be true in general.

Proof: *Given* $A \cap B \underset{C_1+C_2}{\overset{=}{}} A \Rightarrow \underline{O_{C_1+C_2}}(A \cap B) = \underline{O_{C_1+C_2}}(A)$ and

Given $A \cap B \underset{C_1+C_2}{\overset{=}{}} B \Rightarrow \underline{O_{C_1+C_2}}(A \cap B) = \underline{O_{C_1+C_2}}(B)$

From the above two expressions we have

$\underline{O_{C_1+C_2}}(A) = \underline{O_{C_1+C_2}}(B) \Rightarrow$ A $\underset{C_1+C_2}{\overset{=}{}}$ B.

For the converse part logical equivalence of the statements $(a \wedge b) \vee (c \wedge d)$ and $(a \vee c) \wedge (b \vee d)$, where a, b, c and d are any four logical statements. However, from their truth values we find that these two statements are not equivalent to each other in the following case.

a	B	c	d
True	False	False	True
False	True	True	False

So, examples can be provided which satisfy any of the above cases to show that the converse is not true.

(3.3.2) A $\overset{C_1+C_2}{\underset{=}{}}$ B if $A \cup B$ $\overset{C_1+C_2}{\underset{=}{}}$ A and B both. The converse may not be true in general.

Proof: Given $AUB \overset{C_1+C_2}{\underset{=}{}} A \Rightarrow \overline{O_{C_1+C_2}}(AUB) = \overline{O_{C_1+C_2}}(A)$ and

Given $AUB \overset{C_1+C_2}{\underset{=}{}} B \Rightarrow \overline{O_{C_1+C_2}}(AUB) = \overline{O_{C_1+C_2}}(B)$

From the above two expressions we have

$\overline{O_{C_1+C_2}}(A) = \overline{O_{C_1+C_2}}(B) \Rightarrow$ A $\overset{C_1+C_2}{\underset{=}{}}$ B.

The converse part is not true as in property 1. We note that the truth of the converse depends upon the logical equivalence of the two statements, $(a \vee b) \wedge (c \vee d)$ and $(a \wedge c) \vee (b \wedge d)$. However, we find the statements quoted are not true in the following cases.

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a	b	c	d
True	False	False	True
False	True	True	False

So, examples can be constructed such that the above two cases occur to show that the converse part does not hold.

(3.3.3) A $\overset{C_1+C_2}{=}$ A' and B $\overset{C_1+C_2}{=}$ B' may not imply that $A \cup B$ $\overset{C_1+C_2}{=}$ $A' \cup B'$

Proof: The converse part is not true as in property 1. We note that the truth of the converse depends upon the logical equivalence of the two statements, $(a \vee b) \wedge (c \vee d)$ and $(a \wedge c) \vee (b \wedge d)$. However, we find the statements quoted are not true in the following cases.

a	B	c	d
True	False	False	True
False	True	True	False

So, examples can be constructed such that the above two cases occur to show that the converse part does not hold.

(3.3.4) A $\underset{C_1+C_2}{=}$ A' and B $\underset{C_1+C_2}{=}$ B' may not imply that $A \cap B$ $\underset{C_1+C_2}{=}$ $A' \cap B'$

Proof: Let us consider the following real life example to prove the above property.

Example 1: Consider the following data table. Let us consider 3 columns of it, such as, Faculty name, Roles and Project Numbers. Roles column specifies different roles each faculty play in the school, such as, Program chair-1, Division chair-2, and Year co-ordinator-3. Project Numbers column specifies number of the project on which faculty works on.

Table:1. Faculty Information

S.No.	Faculty Name	Division	Collection. Experience (yrs)	Distribution Experience (yrs)	Sex	Roles	Project Numbers
1	Alia –x_1	1	2	0	Female	1	1
2	Brinda-x_2	2	1	0	Female	3	2
3	Cris-x_3	1	3	3	Male	2	2
4	Danya-x_4	2	1	1	Male	2	3
5	Esha-x_5	1	3	3	Female	1, 2	2
6	Feroz-x_6	2	3	0	Male	2	1, 3
7	Gokul-x_7	1	1	4	Male	3	4
8	Harsha-x_8	2	2	4	Male	3	4

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Based on roles and project number columns two sets of covers are obtained as given below.

Let $U =$ Set of faculties $= \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and the following two covers $C_1 and\ C_2$, are generated as given below.

U / C_1 = Covers obtained based on roles of faculties $= \{\{x_1, x_5\}, \{x_3, x_4, x_5, x_6\}, \{x_2, x_7, x_8\}\}$

U / C_2 = Covers obtained based on project numbers they work on $= \{\{x_1, x_6\}, \{x_2, x_3, x_5\}, \{x_4, x_6\}, \{x_7, x_8\}\}$,

Interpretation of approximate equalities

Consider subsets $A, B \subseteq U$. Then the lower approximation of any set can be interpreted as a group of people who are certainly part of the committee and the upper approximation of any set can be interpreted as a group of people who are either certainly or possibly be part of the committee.

Two sets A and B are said to be optimistic bottom equivalent to each other with respect to C_1 and C_2 if their lower approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly in A with respect to C_1 or with respect to C_2 is same as the set of faculties who are certainly in B with respect to C_1 or with respect to C_2.

Two sets A and B are said to be optimistic top equivalent to each other with respect to C_1 and C_2 if their upper approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly or possibly be in A with respect to C_1 and with respect to C_2 is same as the set of faculties who are certainly or possibly be in B with respect to C_1 and with respect to C_2.

Table: 2. Table of minimal descriptors generated for C_1 and C_2

Elements / Minimal Descriptors	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$md_{C_1}(x)$	$\{x_1, x_5\}$	$\{x_2, x_7, x_8\}$	$\{x_3, x_4, x_5, x_6\}$	$\{x_3, x_4, x_5, x_6, \}$	$\{x_5\}$	$\{x_3, x_4, x_5, x_6, \}$	$\{x_2, x_7, x_8\}$	$\{x_2, x_7, x_8\}$
$md_{C_2}(x)$	$\{x_1, x_6\}$	$\{x_2, x_3, x_5\}$	$\{x_2, x_3, x_5\}$	$\{x_4, x_6\}$	$\{x_2, x_3, x_5\}$	$\{x_6\}$	$\{x_7, x_8\}$	$\{x_7, x_8\}$

Let $A = \{x_3, x_4, x_5, x_6\}$, $A' = \{x_1, x_3, x_4, x_5, x_6\}$, $B = \{x_5, x_6, x_7, x_8\}$, and $B' = \{x_1, x_5, x_6, x_7, x_8\}$

$$\underline{O_{C_1+C_2}}(A) = \{x_3, x_4, x_5, x_6\}\ and\ \underline{O_{C_1+C_2}}(A') = \{x_3, x_4, x_5, x_6\}$$

$$\underline{O_{C_1+C_2}}(B) = \{x_5, x_6, x_7, x_8\}\ and\ \underline{O_{C_1+C_2}}(B') = \{x_5, x_6, x_7, x_8\}$$

$$A \cap B = \{x_5, x_6\} \quad and \quad A' \cap B' = \{x_1, x_5, x_6\}$$

$$\underline{O_{C_1+C_2}}(A \cap B) = \{x_5, x_6\}\ and\ \underline{O_{C_1+C_2}}(A' \cap B') = \{x_1, x_5, x_6\}$$

$$\underline{O_{C_1+C_2}}(A \cap B) \neq \underline{O_{C_1+C_2}}(A' \cap B').\ Thus\ \ A \cap B \ \overset{\neq}{{}_{C_1+C_2}}\ A' \cap B'$$

(3.3.5) $A \overset{C_1+C_2}{=} B \Rightarrow A \cup B^C \overset{C_1+C_2}{=} U$

Proof: Given $A \overset{C_1+C_2}{=} B$ => $\overline{O_{C_1+C_2}}(A) = \overline{O_{C_1+C_2}}(B)$. But by (2.5.6)

$\overline{O_{C_1+C_2}}(A \cup B) \supseteq \overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(B)$. Thus we have

$\overline{O_{C_1+C_2}}(A \cup B^C) \supseteq \overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(B^C)$

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$$= \overline{O_{C_1+C_2}}(B) \cup ((\underline{O_{C_1+C_2}}(B))^C) = \overline{O_{C_1+C_2}}(B) \cup ((\overline{O_{C_1+C_2}}B \setminus BN_{C_1+C_2}B)^C)$$

$$= \overline{O_{C_1+C_2}}(B) \cup \left(\overline{O_{C_1+C_2}}(B)\right)^C = U \quad \Rightarrow \quad A \cup B^c \overset{C_1+C_2}{=} U$$

This completes the proof.

(3.3.6) A $\underset{C_1+C_2}{=}$ B => $A \cap B^c \overset{C_1+C_2}{=} \phi$

Proof: Given A $\underset{C_1+C_2}{=}$ B => $\underline{O_{C_1+C_2}}(A) = \underline{O_{C_1+C_2}}(B)$. But by (2.5.7)

$\underline{O_{C_1+C_2}}(A \cap B) \subseteq \underline{O_{C_1+C_2}}(A) \cap \underline{O_{C_1+C_2}}(B)$. Thus we have

$\underline{O_{C_1+C_2}}(A \cap B^C) \subseteq \underline{O_{C_1+C_2}}(A) \cap \underline{O_{C_1+C_2}}(B^C) = \underline{O_{C_1+C_2}}(A) \cap \left(\overline{O_{C_1+C_2}}(B)\right)^C$

$= \underline{O_{C_1+C_2}}(A) \cap (U - \underline{O_{C_1+C_2}}(B) - BN_{C_1+C_2}(B)) \subseteq \underline{O_{C_1+C_2}}(A) \cap (U - \underline{O_{C_1+C_2}}(B)) = \phi$.

=> $A \cap B^c \overset{C_1+C_2}{=} \phi$

(3.3.7)If $A \subseteq B$ and B $\overset{C_1+C_2}{=} \phi$ then A $\overset{C_1+C_2}{=} \phi$

Proof: Given $A \subseteq B$ and B $\overset{C_1+C_2}{=} \phi$. So we have $\overline{O_{C_1+C_2}}(B) = \phi$. As $A \subseteq B$ =>

=> $\overline{O_{C_1+C_2}}(A) \subseteq \overline{O_{C_1+C_2}}(B) = \phi$ => A $\overset{C_1+C_2}{=} \phi$.

(3.3.8) If $A \subseteq B$ and A $\overset{C_1+C_2}{=}$ U then B $\overset{C_1+C_2}{=}$ U

Proof: Given $A \subseteq B$ and A $\overset{C_1+C_2}{=}$ U. So we have $\overline{O_{C_1+C_2}}(A) = U$. As $B \supseteq A$ =>

=> $\overline{O_{C_1+C_2}}(B) \supseteq \overline{O_{C_1+C_2}}(A) = U$ => B $\overset{C_1+C_2}{=}$ U.

(3.3.9) A $\overset{C_1+C_2}{=}$ B iff $A^C \underset{C_1+C_2}{=} B^C$

Proof: Given A $\overset{C_1+C_2}{=}$ B $\overline{O_{C_1+C_2}}(A) = \overline{O_{C_1+C_2}}(B)$

But we know that

$\overline{O_{C_1+C_2}}(A) = (\underline{O_{C_1+C_2}}(A^C))^c \Leftrightarrow (\underline{O_{C_1+C_2}}(A^C))^c = (\underline{O_{C_1+C_2}}(B^C))^C \Leftrightarrow \underline{O_{C_1+C_2}}(A^C) = \underline{O_{C_1+C_2}}(B^C)$

<=> $A^C \underset{C_1+C_2}{=} B^C$

(3.3.10) If A $\underset{C_1+C_2}{=} \phi$ or B $\underset{C_1+C_2}{=} \phi$ then $A \cap B \underset{C_1+C_2}{=} \phi$

Proof: Given A $\underset{C_1+C_2}{=} \phi$ or B $\underset{C_1+C_2}{=} \phi$ => $\underline{O_{C_1+C_2}}(A) = \phi$ or $\underline{O_{C_1+C_2}}(B) = \phi$

=> $\underline{O_{C_1+C_2}}(A) \cap \underline{O_{C_1+C_2}}(B) = \phi$. But by (2.5.7)

$\underline{O_{C_1+C_2}}(A \cap B) \subseteq \underline{O_{C_1+C_2}}(A) \cap \underline{O_{C_1+C_2}}(B)$ => $\underline{O_{C_1+C_2}}(A \cap B) = \phi$ => $A \cap B \underset{C_1+C_2}{=} \phi$.

(3.3.11) If A $\overset{C_1+C_2}{=}$ U or B $\overset{C_1+C_2}{=}$ U then $A \cup B \overset{C_1+C_2}{=}$ U

Proof: Given A $\overset{C_1+C_2}{=}$ U or B $\overset{C_1+C_2}{=}$ U => $\overline{O_{C_1+C_2}}(A) = U$ or $\overline{O_{C_1+C_2}}(B) = U$

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=> $\overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(B) = U$. But by (2.5.6)

$\overline{O_{C_1+C_2}}(A \cup B) \supseteq \overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(B)$ $\overline{O_{C_1+C_2}}(A \cup B) = U$

=> $A \cup B \overset{C_1+C_2}{=} U$.

Replacement properties for first type of covering based optimistic multi granular approximate equalities

These properties are also called as interchange properties. We have stated above the observation of Novotny and Pawlak in connection with holding of the properties for rough equalities when the bottom and top equalities are interchanged. They categorically told that the properties do not hold under this change. However, it is shown by Tripathy et al [16] that some of these properties hold under the interchange where as some other hold with some additional conditions which are sufficient but not necessary. They are stated as below along with their proofs. We use a real life example as detailed below, which shall be used to illustrate the properties as well as provide counter examples whenever necessary.

Example 2: Let us consider **Table–1**. Assume that an exam committee for the school of computing science and engineering (SCSE) is to be constituted to carry out activities such as collecting the question paper bundles and distributing the answer sheet bundles. Assume that there are 8 faculties available for the purpose. Their collection and distribution experiences in years along with their sex and division they belong to are considered for forming two sets of covers as given below.

C_1=Cover obtained by defining similarity relation between two faculties such that they are related to each other iff they belong to different divisions with their average collection experience as exactly 2 years and at most one them be a female faculty

$U/C_1 = \{\{x_1, x_8\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_4, x_5\}, \{x_6, x_7\}\}$

C_2= Cover obtained by defining similarity relation between two faculties such that they are related to each other iff they belong to different divisions with their average distribution experience as exactly 2 years and at most one of them be a female faculty

$U/C_2 = \{\{x_1, x_8\}, \{x_2, x_7\}, \{x_3, x_4\}, \{x_4, x_5\}, \{x_6, x_7\}\}$

Interpretation of approximate equalities

Consider subsets $A, B \subseteq U$. Then the lower approximation of any set can be interpreted as a group of people who are certainly part of the committee and the upper approximation of any set can be interpreted as a group of people who are either certainly or possibly be part of the committee.

Two sets A and B are said to be optimistic bottom equivalent to each other with respect to C_1 and C_2 if their lower approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly in A with respect to C_1 or with respect to C_2 is same as the set of faculties who are certainly in B with respect to C_1 or with respect to C_2.

Two sets A and B are said to be optimistic top equivalent to each other with respect to C_1 and C_2 if their upper approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly or possibly be in A with respect to C_1 and with respect to C_2 is same as the set of faculties who are certainly or possibly be in B with respect to C_1 and with respect to C_2.

Let us consider first type of CBOMGRS. Its lower and upper approximations are determined based on minimal descriptors.

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The minimal descriptor table for the two covers for the above example is as shown below.

Table: 3. Table of minimal descriptors for C_1 and C_2

Elements Minimum Descriptors	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$md_{C_1}(x)$	$\{x_1, x_8\}$	$\{x_2,x_3\}$	$\{x_3\}$	$\{x_4\}$	$\{x_4,x_5\}$	$\{x_6,x_7\}$	$\{x_6,x_7\}$	$\{x_1,x_8\}$
$md_{C_2}(x)$	$\{x_1, x_8\}$	$\{x_2,x_7\}$	$\{x_3,x_4\}$	$\{x_4\}$	$\{x_4,x_5\}$	$\{x_6,x_7\}$	$\{x_6,x_7\}$	$\{x_1,x_8\}$

(3.4.1) A $\overset{C_1+C_2}{=}$ B if $A \cap B \overset{C_1+C_2}{=}$ A and B both. The converse need not be true.

Proof: $Given\ A \cap B \overset{C_1+C_2}{=} A \Rightarrow \overline{O_{C_1+C_2}}(A \cap B) = \overline{O_{C_1+C_2}}(A)$

$Given\ A \cap B \overset{C_1+C_2}{=} B \Rightarrow \overline{O_{C_1+C_2}}(A \cap B) = \overline{O_{C_1+C_2}}(B)$

From the above two expressions we have, $\overline{O_{C_1+C_2}}(A) = \overline{O_{C_1+C_2}}(B) \Rightarrow$ A $\overset{C_1+C_2}{=}$ B.

This part can be interpreted as, if the set of faculty certainly or possibly be in committee for $A \cap B$ with respect to C_1+C_2 is the same as that of A and B, then the set of faculty certainly or possibly be in committee for A with respect to C_1+C_2 will be same as that of B. It means that a committee obtained through common people from sets A and B having same group of people who are either certainly or possibly be in the committee is the same as the committees obtained from A and B having same group of people who are either certainly or possibly be in the committee, then those committees obtained from A and B that way will be equal.

The following example shows that converse need not be true.

Let $A = \{x_3, x_6\}$, $B = \{x_3, x_7\}$ *and*

$\overline{O_{C_1+C_2}}(A) = \{x_3, x_6, x_7\}$, $\overline{O_{C_1+C_2}}(B) = \{x_3, x_6, x_7\}$ *and* $\overline{O_{C_1+C_2}}(A \cap B) = \{x_3\}$

Thus $\overline{O_{C_1+C_2}}(A \cap B) \neq \overline{O_{C_1+C_2}}(A)$ *and* $\overline{O_{C_1+C_2}}(A \cap B) \neq \overline{O_{C_1+C_2}}(B)$

though $\overline{O_{C_1+C_2}}(A) = \overline{O_{C_1+C_2}}(B)$

The converse part can be interpreted as, though the sets of faculty certainly or possibly be in committee with respect to C_1+C_2 are the same for A and B but the set of faculty certainly or possibly be in committee for $A \cap B$ with respect to C_1+C_2 is not same as that of A and B. It means that a committee obtained through common people from sets A and B having same group of people who are either certainly or possibly be in the committee , may not be the same as the committee obtained from A and B having same group of people who are either certainly or possibly be in the committee. Then the committees obtained from A and B that way need not be equal.

(3.4.2)A $\underset{C_1+C_2}{=}$ B if $A \cup B \underset{C_1+C_2}{=}$ A and B both. The converse need not be true

Proof: $Given\ A \cup B \underset{C_1+C_2}{=} A \Rightarrow \underline{O_{C_1+C_2}}(A \cup B) = \underline{O_{C_1+C_2}}(A)$ and

$Given\ A \cup B \underset{C_1+C_2}{=} B \Rightarrow \underline{O_{C_1+C_2}}(A \cup B) = \underline{O_{C_1+C_2}}(B)$

From the above two expressions we have

$\underline{O_{C_1+C_2}}(A) = \underline{O_{C_1+C_2}}(B)$ *and so* $A \underset{C_1+C_2}{=} B$.

This part can be interpreted as, if the set of faculty certainly be in committee for $A \cup B$ with respect to C_1+C_2 is the same as that of A and B, then the set of faculty certainly be in committee for A with respect to C_1+C_2 will be same as that of B. It means that a committee obtained through the people from sets A and B having same group of people who are certainly be in the committee is the same as the committees obtained from A and B having same

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group of people who are certainly be in the committee, then the committees obtained from A and B that way will be equal.

The following example shows that the converse need not be true.

Let $A=\{x_4,x_6\}, B=\{x_4,x_7\}$ and $A\cup B=\{x_4,x_6,x_7\}$

$\underline{O_{C_1+C_2}}(A)=\{x_4\}$ and $\underline{O_{C_1+C_2}}(B)=\{x_4\}$ $\underline{O_{C_1+C_2}}(A\cup B)=\{x_4,x_6,x_7\}$

Thus $\underline{O_{C_1+C_2}}(A\cup B)\neq\underline{O_{C_1+C_2}}(A)$ *and* $\underline{O_{C_1+C_2}}(A\cup B)\neq\underline{O_{C_1+C_2}}(B)$

though $\underline{O_{C_1+C_2}}(A)=\underline{O_{C_1+C_2}}(B)$

The converse part can be interpreted as, though the sets of faculty certainly in committee with respect to C_1+C_2 are the same for A and B, but the set of faculty certainly in committee for $A\cup B$ with respect to C_1+C_2 is not the same as that of A and B It means that a committee obtained through common people from sets A and B having same group of people who are certainly in the committee , may not be the same as the committee obtained from A and B having same group of people who are certainly in the committee. Then the committees obtained from A and B that way need not be equal.

(3.4.3) $A \underset{C_1+C_2}{\overset{=}{}} A'$ and $B \underset{C_1+C_2}{\overset{=}{}} B'$ may not imply that $A\cup B \underset{C_1+C_2}{\overset{=}{}} A'\cup B'$

Proof: The following example establishes the above proof.

Let $A=\{x_3,x_6\}$, $A'=\{x_1, x_3\}$, $B=\{x_6,x_8\}$ and $B'=\{x_7, x_8\}$. Then

$\underline{O_{C_1+C_2}}(A)=\{x_3\}$ *and* $\underline{O_{C_1+C_2}}(A')=\{x_3\}$

$\underline{O_{C_1+C_2}}(B)=\phi$ *and* $\underline{O_{C_1+C_2}}(B')=\phi$

$=>\underline{O_{C_1+C_2}}(A)=\underline{O_{C_1+C_2}}(A')$ *and* $\underline{O_{C_1+C_2}}(B)=\underline{O_{C_1+C_2}}(B')$

Thus $A \underset{C_1+C_2}{\overset{=}{}} A'$ and $B \underset{C_1+C_2}{\overset{=}{}} B'$.

Now, $A\cup B=\{x_3,x_6,x_8\}$, so $\underline{O_{C_1+C_2}}(A\cup B)=\{x_3\}$

Also, $A'\cup B'=\{x_1,x_3,x_7,x_8\}$, so $\underline{O_{C_1+C_2}}(A'\cup B')=\{x_1,x_8\}$

Thus $\underline{O_{C_1+C_2}}(A\cup B)\neq\underline{O_{C_1+C_2}}(A'\cup B').=>A\cup B \underset{C_1+C_2}{\overset{\neq}{}} A'\cup B'$.

(3.4.4) $A \overset{C_1+C_2}{\underset{=}{}} A'$ and $B \overset{C_1+C_2}{\underset{=}{}} B'$ may not imply that $A\cap B \overset{C_1+C_2}{\underset{=}{}} A'\cap B'$

Proof: The following example establishes the proof.

Let $A=\{x_3,x_4\}$, $A'=\{x_3, x_5\}$, $B=\{x_4,x_6\}$ and $B'=\{x_4, x_7\}$. Then

$\overline{O_{C_1+C_2}}(A)=\{x_3,x_4,x_5\}$ *and* $\overline{O_{C_1+C_2}}(A')=\{x_3,x_4,x_5\}$;

$\overline{O_{C_1+C_2}}(B)=\{x_4,x_6,x_7\}$ *and* $\overline{O_{C_1+C_2}}(B')=\{x_4,x_6,x_7\}$.

So, $\overline{O_{C_1+C_2}}(A)=\overline{O_{C_1+C_2}}(A')$ *and* $\overline{O_{C_1+C_2}}(B)=\overline{O_{C_1+C_2}}(B')$

But $A\cap B=\{x_4\}=>\overline{O_{C_1+C_2}}(A\cap B)=\{x_4\}$

and $A'\cap B'=\phi=>\overline{O_{C_1+C_2}}(A'\cap B')=\phi$.

Thus, $\overline{O_{C_1+C_2}}(A\cap B)\neq\overline{O_{C_1+C_2}}(A'\cap B')=>A\cap B \overset{C_1+C_2}{\underset{\neq}{}} A'\cap B'$.

(3.4.5) $A \underset{C_1+C_2}{\overset{=}{}} B => A\cup B^C \underset{C_1+C_2}{\overset{=}{}} U$

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Proof: Given A $\underset{C_1+C_2}{=}$ B => $\underline{O_{C_1+C_2}}(A)=\underline{O_{C_1+C_2}}(B)$. But from (2.5.5) $\underline{O_{C_1+C_2}}(A\cup B)\supseteq\underline{O_{C_1+C_2}}(A)\cup\underline{O_{C_1+C_2}}(B)$.

Thus, we have

$\underline{O_{C_1+C_2}}(A\cup B)\supseteq\underline{O_{C_1+C_2}}(A)\cup\underline{O_{C_1+C_2}}(B)$ = $\underline{O_{C_1+C_2}}(B)\cup(\overline{O_{C_1+C_2}}(B))^c$

= $\underline{O_{C_1+C_2}}(B)\cup(U-(\underline{O_{C_1+C_2}}(B)-BN_{C_1+C_2}B))$

$\subseteq\underline{O_{C_1+C_2}}(B)\cup(U-\underline{O_{C_1+C_2}}(B))=U$ => $A\cup B'$ $\underset{C_1+C_2}{=}$ U

(3.4.6) A $\overset{C_1+C_2}{=}$ B => $A\cap B^c$ $\underset{C_1+C_2}{=}$ ϕ

Proof: Given A $\overset{C_1+C_2}{=}$ B => $\overline{O_{C_1+C_2}}(A)=\overline{O_{C_1+C_2}}(B)$. But from (2.5.8)

$\overline{O_{C_1+C_2}}(A\cap B)\subseteq\overline{O_{C_1+C_2}}(A)\cap\overline{O_{C_1+C_2}}(B)$. Thus, we have

$\overline{O_{C_1+C_2}}(A\cap B^c)\subseteq\overline{O_{C_1+C_2}}(A)\cap\overline{O_{C_1+C_2}}(B^c)=\overline{O_{C_1+C_2}}(A)\cap(\underline{O_{C_1+C_2}}(B))^c=\overline{O_{C_1+C_2}}(A)\cap(U-\underline{O_{C_1+C_2}}(B))$

$\subseteq BN_{C_1+C_2}(B)$

=> $A\cap B'$ $\overset{C_1+C_2}{\neq}$ ϕ.

(3.4.7) If $A\subseteq B$ and B $\underset{C_1+C_2}{=}$ ϕ then A $\underset{C_1+C_2}{=}$ ϕ

Proof: Given $A\subseteq B$ and B $\underset{C_1+C_2}{=}$ ϕ. So, we have $\underline{O_{C_1+C_2}}(B)=\phi$.As $A\subseteq B$=> $A=\phi$ => $\underline{O_{C_1+C_2}}(A)=\phi$

=> A $\underset{C_1+C_2}{=}$ ϕ

(3.4.8) If $A\subseteq B$ and A $\underset{C_1+C_2}{=}$ U then B $\underset{C_1+C_2}{=}$ U

Proof: Given $A\subseteq B$ and B $\underset{C_1+C_2}{=}$ U. So $\underline{O_{C_1+C_2}}(A)=U$ .As $A\subseteq B$=> $\underline{O_{C_1+C_2}}(A)\subseteq\underline{O_{C_1+C_2}}(B)$, we have $\underline{O_{C_1+C_2}}(B)=U$

=> B $\underset{C_1+C_2}{=}$ U.

(3.4.9) A $\underset{C_1+C_2}{=}$ B iff A^c $\overset{C_1+C_2}{=}$ B^c

Proof: Given A $\underset{C_1+C_2}{=}$ B

But we know that $\underline{O_{C_1+C_2}}(A)=(\overline{O_{C_1+C_2}}(A^c))^c$

=>$(\overline{O_{C_1+C_2}}(A^c))^c=(\overline{O_{C_1+C_2}}(B^c))^c$ => $\overline{O_{C_1+C_2}}(A^c)=\overline{O_{C_1+C_2}}(A^c)$

$\underline{O_{C_1+C_2}}(A)=\underline{O_{C_1+C_2}}(B)$ => A^c $\overset{C_1+C_2}{=}$ B^c

In a similar way converse will also be proved.

(3.4.10) If A $\overset{C_1+C_2}{=}$ ϕ or B $\overset{C_1+C_2}{=}$ ϕ then $A\cap B$ $\overset{C_1+C_2}{=}$ ϕ

Proof: Given A $\overset{C_1+C_2}{=}$ ϕ or B $\overset{C_1+C_2}{=}$ ϕ.So, $\overline{O_{C_1+C_2}}(A)=\phi$ *or* $\overline{O_{C_1+C_2}}(B)=\phi$ => $\overline{O_{C_1+C_2}}(A)\cap\overline{O_{C_1+C_2}}(B)=\phi$.

But from (2.5.8) $\overline{O_{C_1+C_2}}(A\cap B)\subseteq\overline{O_{C_1+C_2}}(A)\cap\overline{O_{C_1+C_2}}(B)$ => $\overline{O_{C_1+C_2}}(A\cap B)=\phi$ => $A\cap B$ $\overset{C_1+C_2}{=}$ ϕ

(3.4.11) If A $\underset{C_1+C_2}{=}$ U or B $\underset{C_1+C_2}{=}$ U then $A\cup B$ $\underset{C_1+C_2}{=}$ U

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Proof: Given A $\underset{=}{C_1+C_2}$ U or B $\underset{=}{C_1+C_2}$ U. So, $\underline{O_{C_1+C_2}}(A) = U$ *or* $\underline{O_{C_1+C_2}}(B) = U \;\; \Rightarrow \underline{O_{C_1+C_2}}(A) \cup \underline{O_{C_1+C_2}}(B) = U$.

But from (2.5.7) $\underline{O_{C_1+C_2}}(A \cup B) \supseteq \underline{O_{C_1+C_2}}(A) \cup \underline{O_{C_1+C_2}}(B) \Rightarrow \underline{O_{C_1+C_2}}(A \cup B) = U \Rightarrow A \cup B \;\underset{C_1+C_2}{=}\;$ U.

In a similar manner we can prove the necessity condition.

Approximate rough equivalence for covering based optimistic multi granulation

We now introduce the different rough equivalences for the first type of covering based optimistic multi granular rough set (CBOMGRS). These definitions will be the same for other types. Then we study, prove and provide counter examples for its direct and replacement properties as per requirement.

Let C_1 and C_2 be two covers on U and $C_1, C_2 \in C$ *and* $A, B \subseteq U$. Let F denotes first type CBOMGRS.

Definition 3.5: We say that,

(3.5.1) A and B are bottom rough $C_1 + C_2$ equivalent to each other (A $\underset{C_1+C_2}{\approx}$ B) iff

$\underline{O_{C_1+C_2}}(A)$ *and* $\underline{O_{C_1+C_2}}(B)$ *are* ϕ *or not* ϕ *together*.

(3.5.2) A and B are top rough $C_1 + C_2$ equivalent to each other (A $\overset{C_1+C_2}{\approx}$ B) iff

$\overline{O_{C_1+C_2}}(A)$ *and* $\overline{O_{C_1+C_2}}(B)$ *are* U *or not* U *together*.

(3.5.3) A and B are total rough $C_1 + C_2$ equivalent to each other (A *r_C*$_1$*+C*$_2$*_eqv* B) iff

$\underline{O_{C_1+C_2}}(A)$ *and* $\underline{O_{C_1+C_2}}(B)$ *are* ϕ *or not* ϕ *together* and $\overline{O_{C_1+C_2}}(A)$ *and* $\overline{O_{C_1+C_2}}(B)$ *are* U *or* not U *together*.

Following are the generalization of the approximate rough inclusions introduced by Pawlak [5,6] and approximate rough comparisons introduced by Tripathy et al[16]. We define these concepts in the context of first type of covering based
optimistic multi granulation as below.

Definition 3.6:

Let K=(U,R) be a knowledge base and $A, B \subseteq U$ *and* $C_1, C_2 \in C$. Then

(i) We say A is bottom $C_1 + C_2$ rough included in B $(A \underset{C_1+C_2}{\subseteq} B)$ *iff* $\underline{O_{C_1+C_2}}(A) \subseteq \underline{O_{C_1+C_2}}(B)$.

(ii) We say A is bottom $C_1 + C_2$ rough included in B $(A \overset{C_1+C_2}{\subseteq} B)$ *iff* $\overline{O_{C_1+C_2}}(A) \subseteq \overline{O_{C_1+C_2}}(B)$.

(iii) We say A is rough $C_1 + C_2$ included in B *iff* $\underline{O_{C_1+C_2}}(A) \subseteq \underline{O_{C_1+C_2}}(B)$ *and* $\overline{O_{C_1+C_2}}(B) \subseteq \overline{O_{C_1+C_2}}(A)$.

Definition3.7:

Let K=(U,R) be a knowledge base and $A, B \subseteq U$ *and* $C_1, C_2 \in C$. Then

(i) We say that $A, B \subseteq U$ are bottom $C_1 + C_2$ comparable iff $A \underset{C_1+C_2}{\subseteq} B$ *or* $B \underset{C_1+C_2}{\subseteq} A$.

(ii) We say that $A, B \subseteq U$ are top $C_1 + C_2$ comparable iff $A \overset{C_1+C_2}{\subseteq} B$ *or* $B \overset{C_1+C_2}{\subseteq} A$.

(iii) We say that $A, B \subseteq U$ are $C_1 + C_2$ comparable iff A and B are both bottom $C_1 + C_2$ and top $C_1 + C_2$ comparable.

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Properties for covering based optimistic multi granular approximate equivalence

(3.6.1)(i) If $A\cap B \underset{C_1+C_2}{\approx}$ A and $A\cap B \underset{C_1+C_2}{\approx}$ B then A $\underset{C_1+C_2}{\approx} B.$

(ii) The converse of (i) is not necessarily true.

(iii) The converse is true in (iii) if A and B is bottom C_1+C_2 comparable.

(iv) the condition in (iii) is not necessary.

Proof:

$(i)\ \underline{O_{C_1+C_2}}(A\cap B)\ and\ \underline{O_{C_1+C_2}}(A)\ are\ either\ \phi\ or\ not\ \phi\ together\ (given).$

$\underline{O_{C_1+C_2}}(A\cap B)\ and\ \underline{O_{C_1+C_2}}(B)\ are\ either\ \phi\ or\ not\ \phi\ together\ (given).$

$Then\ \underline{O_{C_1+C_2}}(A)\ and\ \underline{O_{C_1+C_2}}(B)\ are\ either\ \phi\ or\ not\ \phi\ together\ (derived).$

$Thus\ A \underset{C_1+C_2}{\approx} B.$

(ii) Continuing with example2 by taking A={x_3} and B={x_6, x_7}, we have

$\underline{O_{c_1+c_2}}(A)=\{x_3\}\neq\phi\ \ and\ \ \underline{O_{c_1+c_2}}(B)=\{x_6,x_7\}\neq\phi => A\underset{C_1+C_2}{\approx} B.$ But $A\cap B=\phi.$ Then

$\underline{O_{C_1+C_2}}(A\cap B)=\phi => A\cap B\ not\ \underset{C_1+C_2}{\approx} A\ and\ B\ both.$

(iii) Even if A and B is bottom C_1+C_2 comparable, we have $\underline{O_{C_1+C_2}}(A\cap B)\subseteq \underline{O_{C_1+C_2}}(A)\ \ or\ \ \underline{O_{C_1+C_2}}(B)$ as the case

may be. So, if both $\underline{O_{C_1+C_2}}(A)$ and $\underline{O_{C_1+C_2}}(B)$ are ϕ, we have $\underline{O_{C_1+C_2}}(A\cap B)=\phi$. But when both are not ϕ, we cannot say the same for

$\underline{O_{C_1+C_2}}(A\cap B)$.

(iv) Continuing with example2 by taking A={x_4, x_5, x_6, x_7} and B={x_1, x_4, x_8}, we have

$\underline{O_{c_1+c_2}}(A)=\{x_3,x_5,x_6,x_7\}\neq\phi, \underline{O_{c_1+c_2}}(B)=\{x_1,x_4,x_8\}\neq\phi => A \underset{C_1+C_2}{\approx} B.$

$Also\ A\ and\ B\ are\ not\ bottom\ C_1+C_2\ comparable.$

$But\ \ A\cap B=\{x_4\}.\ Then\ \underline{O_{C_1+C_2}}(A\cap B)\neq\phi => A\cap B \underset{C_1+C_2}{\approx} A\ \ and\ \ A\cap B \underset{C_1+C_2}{\approx} B$

$though\ A\ and\ B\ are\ not\ bottom\ C_1+C_2\ comparable.$

(3.6.2)(i) If $A\cup B \overset{C_1+C_2}{\approx}$ A and $A\cup B \overset{C_1+C_2}{\approx}$ B then A $\overset{C_1+C_2}{\approx}$ B.

(ii)The converse of (i) is not necessarily true.

(iii) The converse cannot be true even if A and B are top C_1+C_2 comparable.

(iv) The conditions in (iii) is not necessary.

Proof:

$(i)\ \overline{O_{C_1+C_2}}(A\cup B)\ and\ \underline{O_{C_1+C_2}}(A)\ are\ either\ U\ or\ not\ U\ together\ (given).$

$\overline{O_{C_1+C_2}}(A\cup B)\ and\ \underline{O_{C_1+C_2}}(B)\ are\ either\ U\ or\ not\ U\ together\ (given).$

$Then\ \underline{O_{C_1+C_2}}(A)\ and\ \underline{O_{C_1+C_2}}(B)\ are\ either\ U\ or\ not\ U\ together\ (derived).$

$Thus\ A \overset{C_1+C_2}{\approx} B.$

(ii) Continuing with example 2 by taking A={x_1, x_2, x_3, x_4} and B={x_5, x_6, x_7}, we have

$\overline{O_{C_1+C_2}}(A)=\{x_1,x_2,x_3,x_4,x_8\}\neq U\ \ and\ \ \overline{O_{C_1+C_2}}(B)=\{x_5,x_6,x_7\}\neq U => A\overset{C_1+C_2}{\approx} B.$ But

$A\cup B=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}.Then\ \overline{O_{C_1+C_2}}(A\cup B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U$

$=> A\cup B\ not\ \overset{C_1+C_2}{\approx}\ A\ and\ B\ both.$

(iii) Even if A and B is bottom C_1+C_2 comparable, we have $\underline{O_{C_1+C_2}}(A\cup B)\subseteq \underline{O_{C_1+C_2}}(A)\ \ or\ \ \underline{O_{C_1+C_2}}(B)$ as the

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case may be. So, if both $\underline{O_{C_1+C_2}}(A)$ and $\underline{O_{C_1+C_2}}(B)$ are U, we have $\underline{O_{C_1+C_2}}(A\cup B)=U$. But when both are not U,
we cannot say the same for $\underline{O_{C_1+C_2}}(A\cup B)$.

(iv) Continuing with example 2 by taking A={x_1, x_2, x_3, x_4} and B={x_5, x_6}we have

$$\overline{O_{C_1+C_2}}(A)=\{x_1,x_2,x_3,x_4,x_8\}\neq U \ \ and \ \ \overline{O_{C_1+C_2}}(B)=\{x_5,x_6\}\neq U$$

$$=> \overline{O_{C_1+C_2}}(A)\not\subset \overline{O_{C_1+C_2}}(B) \ or \ \overline{O_{C_1+C_2}}(B)\not\subset \overline{O_{C_1+C_2}}(A)$$

$$=> A \ and \ B \ are \ not \ top \ C_1+C_2 \ comparable.$$

$$And \ \ A\cup B=\{x_1,x_2,x_3,x_4\}.Then \ \overline{O_{C_1+C_2}}(A\cup B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_8\}\neq U$$

$$=> A\cup B \ \underset{\approx}{^{C_1+C_2}} \ A \ and \ B \ both \ though \ A \ and \ B \ are \ not \ top \ C_1+C_2 \ comparable.$$

(3.6.3)(i) If A $\underset{\approx}{^{C_1+C_2}}$ A' and B $\underset{\approx}{^{C_1+C_2}}$ B' then it may or may not be true that $A\cup B \ \underset{\approx}{^{C_1+C_2}} \ A'\cup B'$.

(ii) A sufficient condition for the result in (i) to be true is that A and B are top C_1+C_2 comparable and A' and B' are top C_1+C_2 comparable.

(iii) The conditions in (ii) are not necessary for result in (i) to be true.

Proof: (i) The result fails to be true when all $\overline{O_{C_1+C_2}}(A), \overline{O_{C_1+C_2}}(A'), \overline{O_{C_1+C_2}}(B), \ and \ \overline{O_{C_1+C_2}}(B')$ are not U and exactly one of $A\cup B \ and \ A'\cup B' \ is \ U$, then result will fail. The following example shows that.
Continuing with example 2 by taking A={x_1, x_2, x_3, x_4}, A'= {x_5, x_6, x_7}, B={x_1, x_5, x_6, x_7}, and B'={x_3, x_4, x_5}, we have

$\overline{O_{C_1+C_2}}(A)=\{x_1,x_2,x_3,x_4,x_8\}\neq U \ \ and \ \ \overline{O_{C_1+C_2}}(A')=\{x_5,x_6,x_7\}\neq U$. This implies that A $\underset{\approx}{^{C_1+C_2}}$ A'.

$\overline{O_{C_1+C_2}}(B)=\{x_1,x_5,x_6,x_7\}\neq U \ \ and \ \ \overline{O_{C_1+C_2}}(B')=\{x_3,x_4,x_5\}\neq U$. This implies that B $\underset{\approx}{^{C_1+C_2}}$ B'.

$$But \ A\cup B=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} \ and \ A'\cup B'=\{x_1,x_3,x_4,x_5,x_6,x_7\}$$

$$\overline{O_{C_1+C_2}}(A\cup B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U \ \ and \ \ \overline{O_{C_1+C_2}}(A'\cup B')=\{x_1,x_3,x_4,x_5,x_6,x_7,x_8\}\neq U.$$

$$=> A\cup B \ not \ \underset{\approx}{^{C_1+C_2}} \ A'\cup B'.$$

(ii) We have $\overline{O_{C_1+C_2}}(A)\neq U, \overline{O_{C_1+C_2}}(A')\neq U, \overline{O_{C_1+C_2}}(B)\neq U, \ and \ \overline{O_{C_1+C_2}}(B')\neq U$ .So, under the hypothesis,
$\overline{O_{C_1+C_2}}(A\cup B)\supseteq \overline{O_{C_1+C_2}}(A)\cup\overline{O_{C_1+C_2}}(B) = \overline{O_{C_1+C_2}}(A) \ or \ \overline{O_{C_1+C_2}}(B)$, which is not equal to U. Similarly,
$\overline{O_{C_1+C_2}}(A'\cup B')\neq U$ .Hence $A\cup B \ \underset{\approx}{^{C_1+C_2}} \ A'\cup B'$.

(iii) Continuing with example 2 by taking A={x_1, x_2, x_3, x_4}, A'= {x_5, x_6, x_7},B={x_1, x_5, x_6, x_7}and B'={x_2, x_3, x_4, x_5}, we have

$\overline{O_{C_1+C_2}}(A)=\{x_1,x_2,x_3,x_4,x_8\}\neq U \ \ and \ \ \overline{O_{C_1+C_2}}(A')=\{x_5,x_6,x_7\}\neq U$. This implies that A and A' are not top rough comparable.

$\overline{O_{C_1+C_2}}(B)=\{x_1,x_5,x_6,x_7\}\neq U \ \ and \ \ \overline{O_{C_1+C_2}}(B')=\{x_3,x_4,x_5\}\neq U$. This implies that B and B' are not top rough comparable.

$$But \ A\cup B=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} \ and \ A'\cup B'=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}$$

$$\overline{O_{C_1+C_2}}(A\cup B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U \ \ and \ \ \overline{O_{C_1+C_2}}(A'\cup B')=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U$$

$$=> A\cup B \ \underset{\approx}{^{C_1+C_2}} \ A'\cup B'.$$

(3.6.4)(i) If A $\overset{\approx}{_{C_1+C_2}}$ A' and B $\overset{\approx}{_{C_1+C_2}}$ B' then it may or may not be true that $A\cap B \ \overset{\approx}{_{C_1+C_2}} \ A'\cap B'$.

(ii) A sufficient condition for the result in (i) to be true is that A and Bare bottom $C_1 + C_2$ comparable and A' and B'
are bottom $C_1 + C_2$ comparable.
(iii) The conditions in (ii) are not necessary for result in (i) to be true.

Proof: (i) The result fails to be true when all $\underline{O_{C_1+C_2}}(A), \underline{O_{C_1+C_2}}(A'), \underline{O_{C_1+C_2}}(B), \text{ and } \underline{O_{C_1+C_2}}(B')$ are not ϕ and exactly
one of $A \cap B \text{ and } A' \cap B' \text{ is } \phi$, then result will fail. The following example shows that.
Continuing with example 2 by taking A={x_3}, A'= {x_6, x_7}, B={x_4}, and B'={x_4, x_6, x_7}, we have

$\underline{O_{C_1+C_2}}(A) = \{x_3\} \neq \phi \text{ and } \underline{O_{C_1+C_2}}(A') = \{x_6, x_7\} \neq \phi$. This implies that A $\overset{\approx}{C_1+C_2}$ A'.

$\underline{O_{C_1+C_2}}(B) = \{x_4\} \neq \phi \text{ and } \underline{O_{C_1+C_2}}(B') = \{x_6, x_7\} \neq \phi$. This implies that B $\overset{\approx}{C_1+C_2}$ B'.

$But\ A \cap B = \phi \text{ and } A' \cap B' = \{x_6, x_7\}. \underline{O_{C_1+C_2}}(A \cap B) = \phi \text{ and } \underline{O_{C_1+C_2}}(A' \cap B') = \{x_6, x_7\} \neq \phi.$

$\Rightarrow A \cap B \text{ not } \overset{\approx}{C_1+C_2} A' \cap B'.$

(ii) We have $\underline{O_{C_1+C_2}}(A) \neq \phi, \underline{O_{C_1+C_2}}(A') \neq \phi, \underline{O_{C_1+C_2}}(B) \neq \phi, \text{ and } \underline{O_{C_1+C_2}}(B') \neq \phi$. So, under the hypothesis,

$\underline{O_{C_1+C_2}}(A \cap B) \subseteq \underline{O_{C_1+C_2}}(A) \cap \underline{O_{C_1+C_2}}(B) = \underline{O_{C_1+C_2}}(A) \text{ or } \underline{O_{C_1+C_2}}(B) \neq \phi$. Similarly,
$\underline{O_{C_1+C_2}}(A' \cap B') \neq \phi$.

Hence, $A \cap B \overset{\approx}{C_1+C_2} A' \cap B'$.

(iii) Continuing with example 2 by taking A={x_6, x_7}, A'= {x_1, x_8}, B={x_2, x_6, x_7}, and B'={x_1, x_2, x_8}, we have
$\underline{O_{C_1+C_2}}(A) = \{x_6, x_7\} \neq \phi \text{ and } \underline{O_{C_1+C_2}}(A') = \{x_1, x_8\} \neq \phi$. This implies that A and A' are not bottom $C_1 + C_2$
comparable.
$\underline{O_{C_1+C_2}}(B) = \{x_2, x_6, x_7\} \neq \phi \text{ and } \underline{O_{C_1+C_2}}(B') = \{x_1, x_6\} \neq \phi$. This implies that B and B' are not bottom $C_1 + C_2$
comparable.
$But\ A \cap B = \{x_6, x_7\} \text{ and } A' \cap B' = \{x_1, x_8\}\ \underline{O_{C_1+C_2}}(A \cap B) = \{x_6, x_7\} \neq \phi \text{ and } \underline{O_{C_1+C_2}}(A' \cap B') = \{x_1, x_8\} \neq \phi.$
$\Rightarrow A \cap B \overset{\approx}{C_1+C_2} A' \cap B'.$

(3.6.5) (i)A $\overset{C_1+C_2}{\approx}$ B may or may not imply that $(A \cup \sim B) \overset{C_1+C_2}{\approx}$ U.
(ii) A sufficient condition for the result in (i) to hold is that A and B are bottom $C_1 + C_2$ equal.
(iii) The conditions in (ii) are not necessary for the result in (i) to hold.

Proof: (i) The result fails to hold true when $\overline{O_{C_1+C_2}}(A) \neq U, \overline{O_{C_1+C_2}}(B) \neq U$ and still $\overline{O_{C_1+C_2}}(A \cup \sim B) = U$.
(ii) The condition in (ii) is not sufficient as we have
$\overline{O_{C_1+C_2}}(A \cup \sim B) \supseteq \overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(\sim B) = \overline{O_{C_1+C_2}}(A) \cup \overline{O_{C_1+C_2}}(\sim A) \supseteq \overline{O_{C_1+C_2}}(A \cup \sim A) = U$
(iii) Continuing with example 2 by taking A={x_1, x_3, x_4, x_7} and B={x_1, x_8}, we have
$\underline{O_{C_1+C_2}}(A) = \{x_2, x_3, x_4\} \text{ and } \underline{O_{C_1+C_2}}(B) = \{x_1, x_8\} \Rightarrow$ A and B is not bottom rough equal.
~B={x_2, x_3, x_4, x_5, x_6, x_7} and $A \cup \sim B = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
$\overline{O_{C_1+C_2}}(A \cup \sim B) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} = U \Rightarrow A \cup \sim B \overset{C_1+C_2}{\approx} U$.

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(3.6.6) (i) A $\underset{C_1+C_2}{\approx}$ B may or may not imply that $(A \cup \sim B) \underset{C_1+C_2}{\approx}$ U.

(ii) A sufficient condition for the result in (i) to hold is that A and B are top $C_1 + C_2$ equal.

(iii) The conditions in (ii) are not necessary for the result in (i) to hold.

Proof: (i) The result fails to hold true when $\underline{O}_{C_1+C_1}(A) \neq \phi, \underline{O}_{C_1+C_1}(B) \neq \phi$ and still $\underline{O}_{C_1+C_1}(A \cap B) = \phi$.

(ii) The condition in (ii) is not sufficient as we have

$\underline{O}_{C_1+C_2}(A \cap \sim B) \subseteq \underline{O}_{C_1+C_2}(A) \cap \underline{O}_{C_1+C_2}(\sim B) = \underline{O}_{C_1+C_2}(A) \cup \underline{O}_{C_1+C_2}(\sim A) \subseteq \underline{O}_{C_1+C_2}(A \cap \sim A) = \phi$

(iii) Continuing with example 2 by taking A={x_1, x_2, x_5} and B={x_2, x_3, x_4}, we have

$\overline{O}_{C_1+C_2}(A) = \{x_1, x_2, x_5\}$ *and* $\overline{O}_{C_1+C_2}(B) = \{x_2, x_3, x_5\} =>$ A and B is not top rough equal.

~B={x_1, x_5, x_6, x_7, x_8} and $A \cap \sim B = \{x_1, x_5\}$

$\underline{O}_{C_1+C_2}(A \cap \sim b) = \phi => A \cap \sim B \underset{C_1+C_2}{\approx} \phi$.

(3.6.7) If $A \subseteq B$ and B $\underset{C_1+C_2}{\approx} \phi$ then A $\underset{C_1+C_2}{\approx} \phi$.

Proof: As B $\underset{C_1+C_2}{\approx} \phi$, we have $\underline{O}_{C_1+C_2}(B) = \phi$. So, if $A \subseteq B$, $\underline{O}_{C_1+C_2}(A) \subseteq \underline{O}_{C_1+C_2}(B) = \phi$. Thus A $\underset{C_1+C_2}{\approx} \phi$.

(3.6.8) If $A \subseteq B$ and A $\overset{C_1+C_2}{\approx} U$ then B $\overset{C_1+C_2}{\approx} U$.

Proof: As A $\overset{C_1+C_2}{\approx} U$, we have $\overline{O}_{C_1+C_2}(A) = U$. So, if $A \subseteq B$, $\overline{O}_{C_1+C_2}(B) \supseteq \overline{O}_{C_1+C_2}(A) = U$. Thus B $\overset{C_1+C_2}{\approx} U$.

(3.6.9) A $\overset{C_1+C_2}{\approx}$ B iff ~A $\underset{C_1+C_2}{\approx}$ ~B.

Proof: The proof follows from the property, $\underline{O}_{C_1+C_2}(\sim A) = \sim \overline{O}_{C_1+C_2}(A)$.

(3.6.10) A $\underset{C_1+C_2}{\approx} \phi$, B $\underset{C_1+C_2}{\approx} \phi$ implies that $A \cap B \underset{C_1+C_2}{\approx} \phi$.

Proof: The proof follows directly from the fact that under the hypothesis the only possibility is $\underline{O}_{C_1+C_2}(A) = \underline{O}_{C_1+C_2}(B) = \phi$.

(3.6.11) A $\overset{C_1+C_2}{\approx} U$, B $\overset{C_1+C_2}{\approx} U$ implies that $A \cup B \overset{C_1+C_2}{\approx} U$.

Proof: The proof follows directly from the fact that under the hypothesis the only possibility is $\overline{O}_{C_1+C_2}(A) = \overline{O}_{C_1+C_2}(B) = U$.

Replacement properties for covering based optimistic multi granular approximate equivalence

(3.7.1) (i) if $A \cap B \overset{C_1+C_2}{\approx}$ A and $A \cap B \overset{C_1+C_2}{\approx}$ B then A $\overset{C_1+C_2}{\approx}$ B.

(ii) The converse of (i) is not necessarily true.

Proof: (i) Here $\overline{O}_{C_1+C_2}(A)$ and $\overline{O}_{C_1+C_2}(A \cap B)$ are U or not U together and $\overline{O}_{C_1+C_2}(B)$ and $\overline{O}_{C_1+C_2}(A \cap B)$ are U or not U together. Being common, we get $\overline{O}_{C_1+C_2}(A)$ and $\overline{O}_{C_1+C_2}(B)$ are U or not U together. So, A $\overset{C_1+C_2}{\approx}$ B.

(ii) Continuing with example 2 by taking A={x_1, x_2, x_3, x_4, x_6, x_7} and B={ x_2, x_3, x_4, x_6, x_7,x_8}, we have

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$\overline{O_{C_1+C_2}}(A)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U \;\; and \;\; \overline{O_{C_1+C_2}}(B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U => A\overset{C_1+C_2}{\approx}B.$

But $A\cap B=\{x_2,x_3,x_4,x_6,x_7\}.Then\ \overline{O_{C_1+C_2}}(A\cap B)=\{x_2,x_3,x_4,x_5,x_6,x_7\}\neq U$

$=> A\cap B\ not \overset{C_1+C_2}{\approx} A\ and\ B\ both.$

(3.7.2) (i) if $A\cup B \approx_{C_1+C_2}$ A and $A\cap B \approx_{C_1+C_2}$ B then A $\approx_{C_1+C_2}$ B.

(ii) The converse of (i) is not necessarily true.

Proof: (i) Here $\underline{O_{C_1+C_2}}(A)$ and $\underline{O_{C_1+C_2}}(A\cup B)$ are ϕ or not ϕ together and $\underline{O_{C_1+C_2}}(B)$ and $\underline{O_{C_1+C_2}}(A\cup B)$ are

ϕ or not ϕ together. Being common, we get and $\underline{O_{C_1+C_2}}(B)$ are ϕ or not ϕ together. So, A $\overset{C_1+C_2}{\approx}$ B.

(ii) Continuing with example 2 by taking A={x_6} and B={ x_7}we have

$\underline{O_{C_1+C_2}}(A)=\phi \;\; and \;\; \underline{O_{C_1+C_2}}(B)=\phi => A\approx_{C_1+C_2}B.$ But $A\cup B=\{x_6,x_7\}$. Then

$\underline{O_{C_1+C_2}}(A\cup B)=\{x_6,x_7\}\neq\phi => A\cup B\ \ not\ \approx_{C_1+C_2} A\ and\ B\ both.$

(3.7.3) A $\approx_{C_1+C_2}$ A' and B $\approx_{C_1+C_2}$ B' may not necessarily imply that $A\cup B \approx_{C_1+C_2} A'\cup B'$.

Proof: When $\underline{O_{C_1+C_2}}(A),\underline{O_{C_1+C_2}}(B),\underline{O_{C_1+C_2}}(A'),\underline{O_{C_1+C_2}}(B')$ are all ϕ, one of $\underline{O_{C_1+C_2}}(A\cup B)\ and\ \underline{O_{C_1+C_2}}(A'\cup B')$ is ϕ but the other one is not ϕ the result fails to be true. Continuing example2 by taking A={x_6}, A'={x_8},B={x_2}and B'={x_1},
we have

$\underline{O_{C_1+C_2}}(A)=\phi,\underline{O_{C_1+C_2}}(A')=\phi,\underline{O_{C_1+C_2}}(B)=\phi,\ and\ \underline{O_{C_1+C_2}}(B')=\phi..$

$But\ A\cup B=\{x_2,x_6\}\ and\ A'\cup B'=\{x_1,x_8\}$

$\underline{O_{C_1+C_2}}(A\cup B)=\phi\ \ and\ \ \underline{O_{C_1+C_2}}(A'\cup B')=\{x_1,x_8\}\neq\phi.$

$=> A\cup B\ \ not\ \overset{C_1+C_2}{\approx}\ A'\cup B'.$

(3.7.4) A $\overset{C_1+C_2}{\approx}$ A' and B $\overset{C_1+C_2}{\approx}$ B' may not necessarily imply that $A\cap B \overset{C_1+C_2}{\approx} A'\cap B'$.

Proof: When $\overline{O_{C_1+C_2}}(A),\overline{O_{C_1+C_2}}(B),\overline{O_{C_1+C_2}}(A'),\overline{O_{C_1+C_2}}(B')$ are all U, one of $\overline{O_{C_1+C_2}}(A\cap B)\ and\ \overline{O_{C_1+C_2}}(A'\cap B')$
is U but the other one is not U the result fails to be true.

Continuing with example 2 by taking A={x_1, x_2, x_3, x_4, x_5, x_7}, A'= { x_1, x_2, x_3, x_4, x_6, x_7, x_8},B={ x_2, x_3, x_4, x_5, x_7, x_8},

and B'={ x_2, x_3, x_4, x_6, x_7, x_8}, we have

$\overline{O_{C_1+C_2}}(A)=\{x_1,\ x_2,\ x_3,\ x_4,x_5,\ x_6,\ x_7,\ x_8\}=U,\overline{O_{C_1+C_2}}(A')=\{x_1,\ x_2,\ x_3,\ x_4,x_5,\ x_6,\ x_7,\ x_8\}=U,$

$\overline{O_{C_1+C_2}}(B)=\{x_1,\ x_2,\ x_3,\ x_4,x_5,\ x_6,\ x_7,\ x_8\}=U,\ and\ \overline{O_{C_1+C_2}}(B')=\{x_1,\ x_2,\ x_3,\ x_4,x_5,\ x_6,\ x_7,\ x_8\}=U.$

$But\ A\cap B=\{x_2,\ x_3,\ x_4,\ x_5,\ x_7\}\ and\ A'\cap B'=\{x_2,\ x_3,\ x_4,\ x_6,\ x_7,\ x_8\}\ \overline{O_{C_1+C_2}}(A\cap B)=\{x_2,\ x_3,\ x_4,\ x_5,\ x$

$and\ \ \overline{O_{C_1+C_2}}(A'\cap B')=\{x_1,x_2,\ x_3,\ x_4,\ x_5,\ x_7,x_8\}=U => A\cap B\ not\ \approx_{C_1+C_2}\ A'\cap B'.$

(3.7.5) A $\approx_{C_1+C_2}$ B may or may not imply that $A\cup \sim B$ $\approx_{C_1+C_2}$ U.

Proof: Continuing with example 2 by taking A={x_1, x_3, x_4, x_7, x_8} and B={x_1}, we have

$\overline{O_{C_1+C_2}}(A)=\{x_1,x_3,x_4,x_7,x_8\}\ \ and\ \ \overline{O_{C_1+C_2}}(B)=\{x_1,x_8\}.$

~B={x_2, x_3, x_4, x_5, x_6, x_7, x_8} and $A\cup\sim B=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_7\}=U$

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$$\underline{O_{C_1+C_2}}(A\bigcup \sim B)=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}=U \Rightarrow A\bigcup \sim B \overset{C_1+C_2}{\underset{\approx}{}} U.$$

(3.7.6) A $\overset{C_1+C_2}{\underset{\approx}{}}$ B may or may not imply that $A\bigcap \sim B \overset{C_1+C_2}{\underset{\approx}{}} .\phi$.

Proof: Continuing with example 2 by taking A={x_5} and B={x_5, x_6, x_7}, we have

$\underline{O_{C_1+C_2}}(A)=\phi$ *and* $\underline{O_{C_1+C_2}}(B)=\{x_6,x_7\} \Rightarrow$ A and B is not top rough equal.

~B={x_1, x_2, x_3, x_4, x_8} and $A\bigcap \sim B=\phi$

$\underline{O_{C_1+C_2}}(A\bigcap \sim B)=\phi \Rightarrow A\bigcap \sim B \underset{C_1+C_2}{\overset{\approx}{}} \phi.$

We would like to make the following comments in connection with the following properties from (3.7.7) to (3.7.11).

(i) We know that $\underline{O_{C_1+C_2}}(U)=U$. So, bottom C_1+C_2 -equivalent to U can be considered under the case that

$\underline{O_{C_1+C_2}}(U)\neq\phi.$

(ii) We know that $\overline{O_{C_1+C_2}}(\phi)=\phi$. So, bottom C_1+C_2 -equivalent to U can be considered under the case that

$\overline{O_{C_1+C_2}}(\phi)\neq U.$

The proofs of the properties from (3.7.7) to (3.7.11) are trivial and we omit them.

(3.7.7) If $A\subseteq B$ and B $\overset{C_1+C_2}{\underset{\approx}{}}$ ϕ then A $\overset{C_1+C_2}{\underset{\approx}{}}$ ϕ.

(3.7.8) If $A\subseteq B$ and B $\underset{C_1+C_2}{\overset{\approx}{}}$ U then A $\underset{C_1+C_2}{\overset{\approx}{}}$ U.

(3.7.9) A $\underset{C_1+C_2}{\overset{\approx}{}}$ B iff ~A $\underset{C_1+C_2}{\overset{\approx}{}}$ ~B.

(3.7.10) A $\overset{C_1+C_2}{\underset{\approx}{}}$ ϕ, B $\overset{C_1+C_2}{\underset{\approx}{}}$ $\phi \Rightarrow A\bigcap B \overset{C_1+C_2}{\underset{\approx}{}} \phi$.

(3.7.11) A $\underset{C_1+C_2}{\overset{\approx}{}}$ U, B $\underset{C_1+C_2}{\overset{\approx}{}}$ $U \Rightarrow A\bigcup B \overset{C_1+C_2}{\underset{\approx}{}} U$.

CONCLUSION

The equality of sets in mathematics is too stringent and is mostly not applicable in real life situations. The problem in this definition is that although in real life situations we use our knowledge about the universe to decide about the equality of sets, which is mostly approximate in nature, we do not do so for set equality. As an attempt to incorporate user knowledge in equality, Novotny and Pawlak introduced the concept of rough equality and Tripathy et al introduced the concept of rough equivalence. The unigranular rough set concept introduced by Pawlak has been extended to define multigranular rough sets by Qian et al. Also, instead of using partitions, covers have been used to define covering based rough sets recently. In this paper we define and study the rough equality and rough equivalence in the context of covering based optimistic multigranular rough sets and to establish their properties in the general form as well as in the replacement form. We take the help of a real life example to illustrate the concepts and also to provide counter examples is establishing the properties.

CONFLICT OF INTEREST
Authors declare no conflict of interest.

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