

## ARTICLE

# MULTICRITERIA ANALYSIS OF MATHEMATICAL MODELS FOR SOFTWARE RELIABILITY EVALUATION

Vladimir V. Lomakin\*, Natalia P. Putivtseva, Tatyana V. Zaitseva, Olga P. Pusnaya

*Institute of Engineering Technology and Natural Science, Belgorod State National Research University, Belgorod, 85 Pobedy st., 308015, Belgorod, RUSSIA,*

## ABSTRACT

The paper presents the results of an experiment to select the most preferred model for software reliability evaluation. Since the mathematical models used to assess the reliability of software have a number of characteristics that are not the same in importance, it is advisable to apply multi criteria assessment methods to select the most suitable approach to reliability evaluation. A hierarchy analysis was chosen in the capacity of such a method, which allows us to obtain the results of comparing objects in the form of numerical weights. The paper presents the solution to the problem of choosing a dynamic model for evaluating software reliability using classical and simplified hierarchy analysis methods.

## INTRODUCTION

We understand the reliability model of a Platform software as a mathematical model that reflects the dependence of a given software tool reliability on a number of parameters which values are either previously known or can be measured in the process of observing the system operation or in an experimental study of the Platform functioning [1-3].

Depending on the mathematical tools, analytical and empirical reliability models are distinguished. Two groups of the models are allocated depending on the need to consider the time factor in evaluating the reliability: dynamic and static [1-6]. In dynamic models, the behaviour of the software under test over time is considered. In static models, the appearance of failures takes into account only the dependence of the number of errors on the number of test runs or the dependence of the number of errors on the characteristics of the input data.

**Hypothesis:** the Mus's model is the most preferred of the dynamic models for evaluating software reliability.

Within the framework of this study, we restrict ourselves to considering only analytical dynamic models due to the fact that often there is a need to obtain data on the occurrence of failures in time, both continuously and discretely.

Consider the following dynamic reliability models [6]: Shooman's model; La Padula's model; Jelinski-Moranda's model; Chic-Walverton's model; Mus's model; Model of transition probabilities.

**Shooman's model.** The initial data for the Shooman's model is collected during the software testing process. At each time interval, a program is run on a full range of developed test data and a certain number of errors are recorded. Statistics on detected errors are collected. After the end of the stage, the errors found in the previous stage are corrected, test sets are adjusted, if necessary, and a new testing stage is conducted. The reliability function is calculated as the probability of no failure on a time interval from 0 to t.

**La Padula's Model.** According to this model, a sequence of tests is performed in t stages. Each stage ends with the introduction of changes (corrections) to the software. The reliability of the software during the i-th stage is calculated as the difference between the marginal reliability of the software at this stage and the coefficient of the growth parameter. Being based on the data obtained during testing, the model makes it possible to predict the likelihood of a program running smoothly at subsequent stages of its execution.

Initial data for the Jelinski – Moranda's model is collected in the process of testing software. At the same time, the time until the next failure is recorded. Each detected error is resolved. The software reliability is calculated as a function of the i-th error detection time distribution density, counted from the moment when the (i-1) th error was detected.

The Chic-Walverton's model considers that errors are corrected only after the expiration of the time interval at which they arose. The error rate is proportional to the number of errors remaining in the program after the (i-1) th interval and the total time already spent on testing.

**Mus's model.** During testing, the program execution time (test run) until the next failure is recorded. It is allowed to detect more than one error during program execution until the next failure occurs. The total

**KEY WORDS**  
software reliability,  
mathematical model,  
multivariate analysis,  
analytic hierarchy  
process (ahp).

Received: 12 Oct 2019  
Accepted: 24 Nov 2019  
Published: 4 Jan 2020

\*Corresponding Author  
Email:  
lomakin@bsu.edu.ru

number of failures occurred throughout the entire software life cycle is related with the initial number of errors depending on the number of errors eliminated per failure.

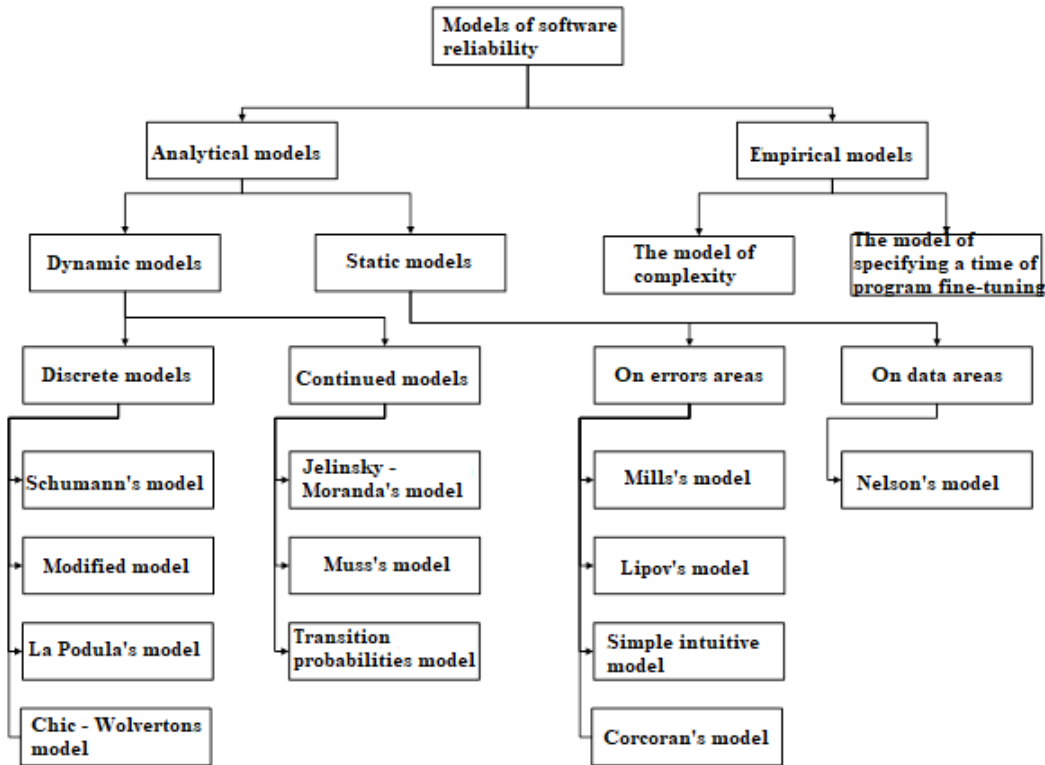


Fig. 1: Classification of reliability models.

The transition probability model is based on a Markov process that takes place in a discrete system with continuous time. It is suggested that during the testing process one error is detected. At any moment in time, the system can be in two possible states - operational or in a moment of another error correction. System readiness is defined as the sum of the probabilities of finding it in an operational state.

## METHODS

When solving problems, the classical analytic hierarchy process (Saaty method) and the simplified method were used.

Consider the steps of applying the classical method [7]:

1. Hierarchical decomposition of the problem "top-down".
2. A comparative assessment of the importance of the hierarchical structure elements in relation to the overlying level based on a unified scale.  
A set of pairwise comparison matrices of the elements  $H_i$  and  $H_j$  of any hierarchical level  $A_k = ||a_{ijk}||_{h \times h}$ ,  $a_{ijk} = s_i/s_j$ , where  $h$  is the number of compared basic elements, and where the preference of elements for decision-making is defined as  $H_i > H_j$  if  $a_{ijk} > 1$ ;  $H_i \approx H_j$  if  $a_{ijk} = 1$ ;  $H_i < H_j$  if  $a_{ijk} < 1$ .
3. Calculation of the priority of options by aggregating particular estimates of the hierarchical structure elements in the direction "bottom-up".

In that case, when the simplified method is used, a simplified procedure is carried out for calculating the relative priorities of the options and criteria when constructing the pairwise comparison matrix  $A_k = ||a_{ijk}||_{h \times h}$  of elements  $H_i$  and  $H_j$  of any hierarchical level [8]. The decision maker selects an element as a reference and assigns it the first number. The remaining elements are given arbitrary numbers. A pairwise comparison of each of the elements  $H_i$ ,  $i = 2, 3, \dots, h$  with the reference  $H_1$  is performed. The element  $H_i$  is associated with a certain positive number  $b_{1i}$  showing the subjective significance of this element for the decision-maker with respect to the element  $H_1$ . As a result, all elements  $H_i$  will be associated with positive numbers  $b_{11}, b_{12}, \dots, b_{1h}$ , where  $b_{11} = 1$ . The sequence of numbers  $b_{11}, b_{12}, \dots, b_{1h}$  is taken as the first row of the square pairwise comparison matrix  $B = ||b_{ij}||_{h \times h}$  of rank  $h$ . The matrix  $B$  is constructed from the very beginning so that its elements  $b_{ij}$  satisfy the conditions of inverse symmetry  $b_{ij} \times b_{ji} = 1$  and compatibility  $b_{ij} \times b_{jk} = b_{ik}$  to satisfy the equalities  $b_{ij} = b_{i1} \times b_{1j} = b_{1j} / b_{1i}$ , for all numbers  $i, j = 2, \dots, h$ , with the help of which all elements of the pairwise comparison matrix  $B$  are obtained in a unique way.

A comparative importance indicator for the hierarchy element  $H_i$  is introduced in the form of  $s_i = b_{i1} / b_{h1} = b_{1h} / b_{1i}$ . Then the local priority of the element  $H_i$  will be determined by the expression  $v_{ik} = v_k(H_i) = s_i / \sum$ . For an arbitrary element  $b_{ij}$  of the matrix  $B$ , the relation  $b_{ij} = s_i / s_j$  is satisfied. The column vector  $s = (s_1, s_2, \dots, s_h)^T$  composed of the values of comparative importance indicators is an eigenvector of the matrix  $B$ , and its eigenvalue is equal to  $h$ . Thus, the matrix  $B$  is consistent.

### IMPLEMENTATION

The following criteria were chosen for solving the problem of choosing an analytical dynamic model: simplicity of calculations; forecasting ability; iteration ability, correction of errors at the time of finding; registration of failure; the average program execution time in a period; ratio of the number of errors eliminated to failures; accumulation of error data; the need for software support. The following dynamic models were considered as alternatives: Shooman's; La Padula's; Jelinski - Moranda's; Chic - Walverton's; Mus's; and transition probabilities model.

When conducting the experiment, the program "System for automated calculating according to the analytic hierarchy process" was used [9]. When working with the tool, at the first step it was necessary to enter the parameters which include the selection criteria and alternative solutions [Fig. 2].

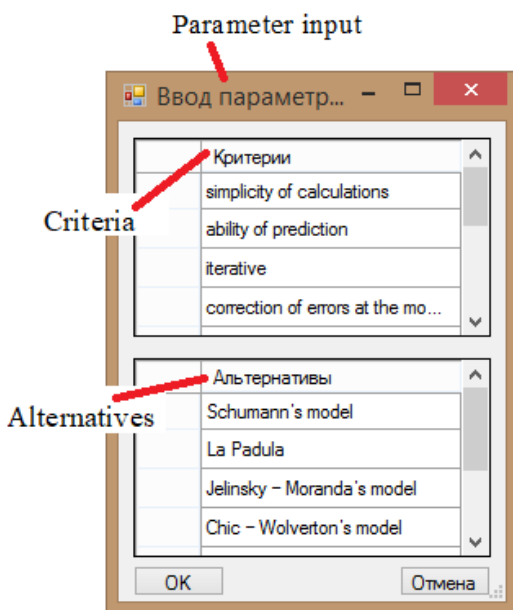


Fig. 2: Entering the decision criteria and their alternatives.

Next, we need to select the method on the basis of which the selection will take place; for the first experiment, the classic Saati's method was chosen [Fig. 3].

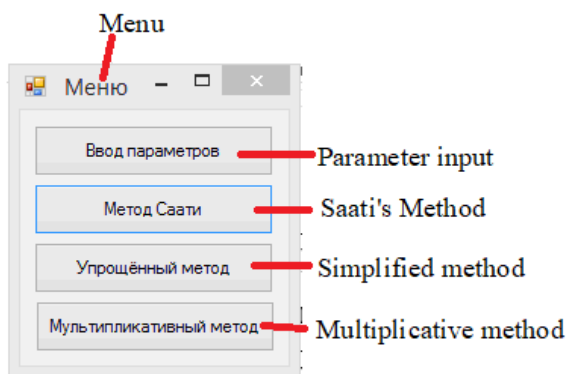


Fig. 3: Choice of a method (Saati method).

In the next step, we fill in the matrix of paired criteria comparisons [Fig. 4].

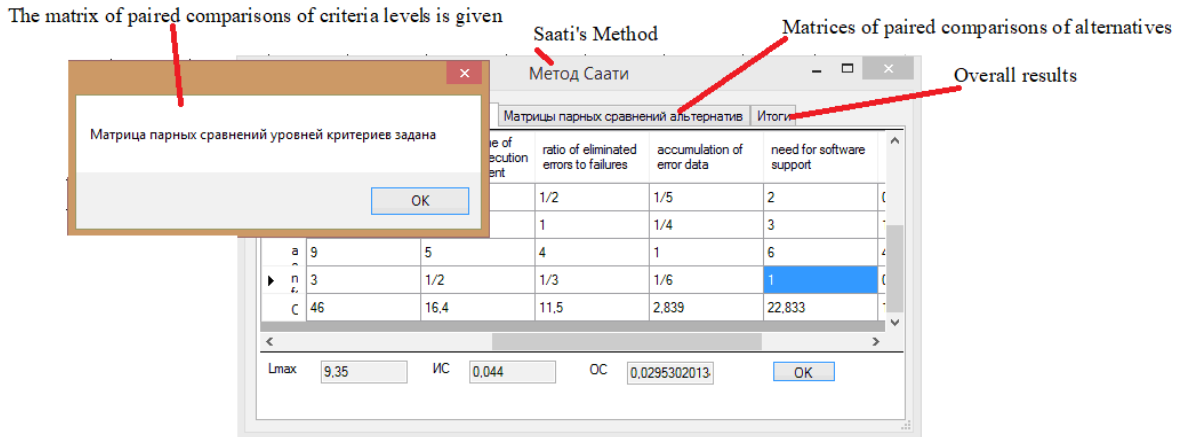


Fig. 4: Filling in the paired criteria comparisons matrix.

Next, the matrices of pairwise alternative comparisons for each criterion are filled. [Fig. 5] shows the result of filling in a pairwise alternative comparison matrix by the criterion of "simplicity of computation". The remaining matrices are filled in the same way.

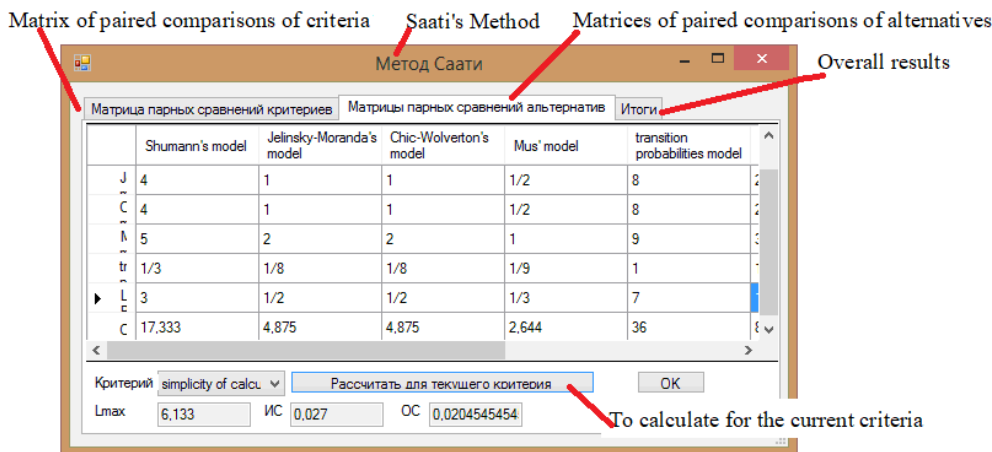


Fig. 5: Filling in the pairwise alternative comparison matrix by the criterion of "ease of calculation".

After filling in all the matrices, the total weights of the alternatives were calculated; the alternatives are shown in [Fig. 6], and all summary data are presented in [Table 1].

When applying the simplified method, we must select the appropriate method in the menu following the stage of filling in the parameters.

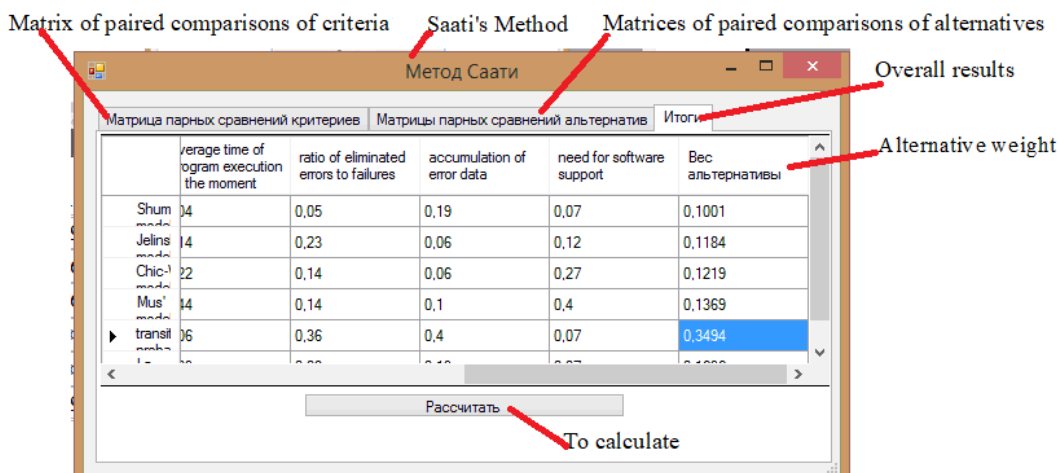


Fig. 6: Calculation of the total weights of the alternatives.

Table 1: Summary (Saati method)

Alternatives	Criteria									Alternative weight
	simplicity of calculations	forecasting ability	Iteration ability	correction of errors at the time of finding	failure record	average program execution time in a period	ratio of the numbers of errors eliminated to failures	error data accumulation	need for software support	
Shooman's Model	0.06	0.05	0.29	0.06	0.04	0.04	0.05	0.19	0,07	0,1001
La Padula's Model	0.22	0.05	0.06	0.22	0.16	0.14	0.23	0.06	0.12	0.1184
Jelinsky – Moranda's Model	0.22	0.05	0.06	0.22	0.25	0.22	0.14	0.06	0.27	0.1219
Chick – Walverton's Model	0.35	0.09	0.11	0,03	0.1	0.44	0.14	0.1	0.4	0.1369
Mus's Model	0,03	0.48	0.18	0.35	0.38	0.06	0.36	0.4	0,07	0.3494
Transition Probability Model	0.13	0.26	0.29	0.13	0.06	0.09	0.08	0.19	0,07	0.1696

In the next step, fill in the first row of the paired criteria comparisons matrix; the remaining rows are filled in automatically [Fig. 8].

Next, the matrices of pairwise alternative comparisons for each criterion are filled. The user fills only the first row of the matrix, and the rest of the values in the rows are calculated by the program. After filling in all the matrices, the total weights of the alternatives which are shown in [Fig. 8] were calculated; and all summary data are presented in [Table 2].

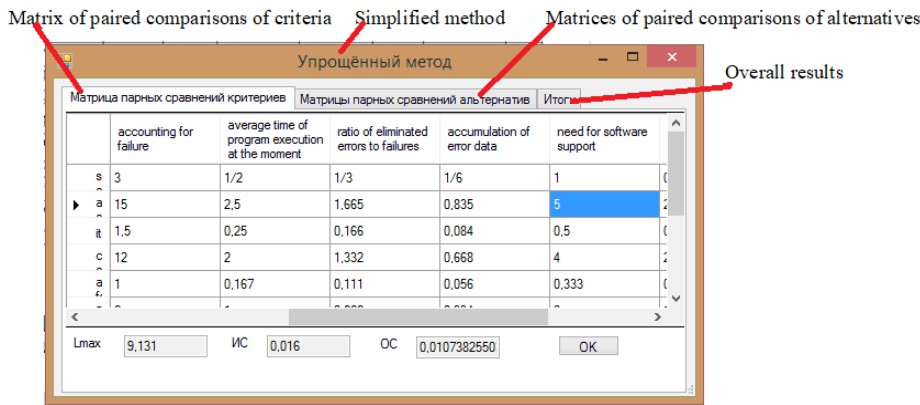


Fig. 7: Filling in the matrix of paired criteria comparisons.

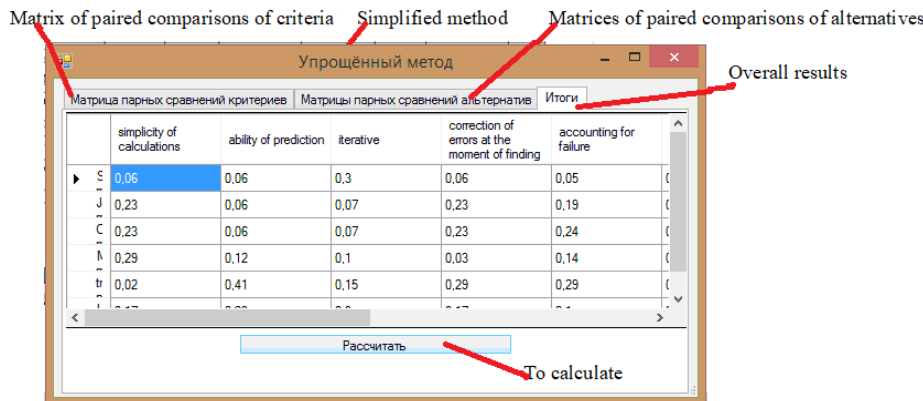


Fig. 8: Calculation of the total weights of the alternatives.

THE IIOAB JOURNAL

**Table 2:** Summary (simplified method)

Alternatives	Criteria									
	simplicity of calculations	forecasting ability	iteration ability	correction of errors at the time of finding	failure record	average program execution time in a period	ratio of the number of errors eliminated to failures	error data accumulation	need for software support	Alternative weight
Shooman's Model	0.06	0.06	0.3	0.06	0.05	0.05	0.05	0.19	0.07	0.096
La Padula's Model	0.23	0.06	0.07	0.23	0.19	0.18	0.22	0.06	0.14	0.133
Jelinsky – Moranda's Model	0.23	0.06	0.07	0.23	0.24	0.23	0.17	0.06	0.29	0.138
Chick – Walverton's Model	0.29	0.12	0.1	0.03	0.14	0.32	0.17	0.1	0.36	0.138
Mus's Model	0.02	0.41	0.15	0.29	0.29	0.09	0.28	0.39	0.07	0.298
Transition Probability Model	0.17	0.29	0.3	0.17	0.1	0.14	0.11	0.19	0.07	0.187

## RESULTS

An analysis of the results obtained using the classical and simplified methods shows that:

- The most important criterion is “the ratio of the number of errors eliminated to failures”, as well as “correction of errors at the time of finding” and “average program execution time in a period”.
- Shooman’s and transition probabilities models are the best alternatives according to the criterion of “iteration ability”.
- The Chick – Walverton’s model is the best alternative according to the criteria: “simplicity of calculations”, “average time of program execution in a period”, and “need for software support”.
- The Mus’s model is the best alternative according to the criteria: “forecasting ability”, “error correction at the time of finding”, “accounting for the presence of a failure”, “ratio of the number of errors eliminated to failures” and “accumulation of error data”.
- The best alternative (with a wide margin from the next - 0.3494 versus 0.1696 for the classical method and 0.2978 versus 0.1873) is the Mus’s Model.

## CONCLUSION

Thus, the proposed hypothesis is confirmed by an experiment for both processes of analytic hierarchy. The Mus’s model for the chosen criteria for selection alternatives is the best of the analytical dynamic models.

However, the obtained results indicate the unambiguous priority of the Mus’s model, from which we can conclude that it is necessary to conduct a practical experiment in which, using the example of one of the Platform modules, to build and evaluate models that have the highest weight values from alternatives, to consider a larger number of alternatives from analytical models, to increase the number of criteria by which alternatives are considered, and to divide all criteria into groups for a more demonstrable picture concerning the influence of criteria on the choice of alternatives.

### CONFLICT OF INTEREST

There is no conflict of interest.

### ACKNOWLEDGEMENTS

Completed as part of implementing a comprehensive project on creation of high-tech production “Development of a methodology and tools for creating applications, supporting the life cycle of information technology and decision-making software for the effective implementation of administrative and management processes within the established authorities”, 2017-218-09-187; Decree of the Government of the Russian Federation dated April 9, 2010 No. 218

### FINANCIAL DISCLOSURE

None.

## REFERENCES

- [1] Gozzi K, et al. [2005] Fundamentals of software engineering, Translated from English SPb.: BHV-Petersburg, RU.
- [2] Orlov SA. [2003] Software development technology: Textbook. St Petersburg, RU.
- [3] Shiryayev MV, Andreeva ON. [2014] The direction of increasing the reliability of special software for embedded systems, Bulletin of MSTU MIREA, 2(3):176-183 [https://rtj.mirea.ru/upload/medialibrary/9c3/16-shiryayev\\_andreeva.pdf](https://rtj.mirea.ru/upload/medialibrary/9c3/16-shiryayev_andreeva.pdf)
- [4] Lyubitsyn VN. [2012] The need to develop reliable software as a challenge of our time. Bulletin of SUSU. Series Computer technology, management, electronics. 16(23):26-29.
- [5] Pakulin NV, Lavrishcheva EM, Ryzhov AG, Zelenov SV. [2018] Analysis of methods for assessing the reliability of equipment and systems. The practice of applying methods. Proceedings of the Institute for System Programming of the Russian Academy of Sciences, 30(3):99-120. [https://doi.org/10.15514/ISPRAS-2018-30\(3\)-8](https://doi.org/10.15514/ISPRAS-2018-30(3)-8)
- [6] Vasilenko NV, Makarov VA. [2004] Models for assessing the reliability of software, Bulletin of the Novgorod state university, 28:126 - 132.
- [7] Saati TL. [2008] Decision making with dependencies and feedbacks: Analytical networks. Translated from English, 360.
- [8] Nogin VD. [2004] A simplified version of the hierarchy analysis method based on nonlinear convolution of criteria, Journal of Computational Mathematics and Mathematical Physics, 44(7):1261-1270; Comput Math And Math Phys, 44(7):1194-1202
- [9] Automation system for calculating acc. to the hierarchy analysis method: Certificate on registration of databases, computer programs No. 2018615850 dated 17/05/2018.