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SPECIAL ARTICLE

MATHEMATICAL APPROACH TOWARDS RECENT INNOVATION IN COMPUTATION AND ENGINEERING SYSTEM (MATRICS)

A CHARACTERIZATION ON GRACEFUL LABELING OF COMPLEMENT OF CAYLEY DIGRAPH

Rajeswari R.*, Manjula T., Parameswari R.

Sathyabama Institute of Science and Technology, Chennai-119, INDIA

ABSTRACT

A digraph is said to be edge graceful if there exists a bijection f: E → { 1, 2, ..., |E|} such that the induced mapping f: V → { 1, 2, ..., |V|-1} given by f*(vi) = ∑ f (eij) (mod |V|) taken over all the outgoing arcs of vi is a bijection where eij is the jth outgoing arc from the vertex vi. In this paper the graceful labeling of complement of Cayley digraph with n vertices and m generators and its line digraph were studied and obtained a characterization for the above graph to admit graceful labeling with respect to antimagic labeling .*

KEY WORDS
Cayley Digraph, Complement of a Graph, Edge Graceful labeling, Antimagic labeling

INTRODUCTION

Rosa introduced β valuation for a graph and later it was termed as graceful labeling. Various research papers were published on the topic of graceful labelings. If the communication grid is a graceful graph, we would then be able to label the connections between each centres such that each connection would have a distinct label.

Bloom and Hsu extended the concept of graceful labeling for digraphs and characterized algebraic structures based on the results. Further the relationship of graceful digraphs with other algebraic structures and the applications of graceful digraphs in networking were also discussed in Bloom and Hsu"s paper.

Being the next development, labelings on vertex symmetric digraphs were studied by Thirusangu et.al. [1]. labelings of special class of digraphs called quadratic residue digraph was studied in [2, 6, 7]. The necessary condition for a digraph to be edge graceful is q(q+1) ≡ 0 or (p/2) (mod p). We always know that the Cayley digraph has mp arcs and hence q(q+1) = mp(mp+1) ≡ 0 (mod p). Some theories of graceful labeling and magic labeling on Cayley digraphs were studied in [3][4][5]. The complement of Cayley digraph C.Cay(G,S) is a digraph on the same vertices such that two distinct vertices of C.Cay(G,S) are adjacent if and only if they are not adjacent in Cay(G,S)[8].

Line digraphs are fascinating structures in the study of dense digraphs.This paper characterizes some graceful technologies on the Complement of Cayley digraph and its line digraph.

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CASE STUDIES

Edge graceful labeling

A digraph is said to be edge graceful if there exists a bijection bijection f: E → { 1, 2, 3, ..., |E|} such that the induced mapping f*:V→{0, 1, 2, 3, ..., |V|-1} given by $f^*(v_i) = \sum_{i=1}^{q} f(e_{ij}) (\bmod |V|)$ taken over all the outgoing arcs of v_i, 1≤ i ≤ |V| is a bijection where e_{ij} is the jth outgoing arc of the vertex v_i.

Theorem

The Complement of Cayley digraph C.Cay(G,S) with generating set S admits Edge graceful labeling if | S | ≡ 0(mod2).

Proof: Consider the Cayley digraph with n vertices, m generators, m ≡ 0(mod2). The complement of Cayley digraph has n vertices and nq arcs where q = n - m-1. Denote the vertex set of C.Cay(G,S) as V= $\{v_1, v_2, v_3, \ldots v_n\}$ and the edge set of C.Cay(G,S) as E= $\{e_{ij}, 1 \le i \le n, 1 \le j \le q\}$. To prove that the complement of Cayley digraph admits edge graceful labeling, we have to show there exists a bijection f : E→{1, 2, 3, ..., |E|} such that the induced mapping f*:V→{0, 1, 2, 3, ..., |V|-1}given $f^*(v_i) = \sum_{i=1}^{q} f(e_{ij}) (\bmod |V|)$) is a bijection.

Define f : E →{ 1, 2, 3, ..., |E|} as

***Corresponding Author**
Email: rajeswarivel1998@gmail.com
Tel.: 9444126406

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$f(e_{ij})$ = (j-1)n+i if j is odd
jn+1-i if j is even

Then the induced function

$f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$
= i +(q-1)/2 +(n(q^2-1))/2
= i +(q-1)/2 (mod n)

Since q is constant, $f^*(v_i)$ is distinct for every i, 1≤ i ≤ n. Hence the C.Cay (G,S) is edge graceful if | S | ≡ 0(mod2).

Corollary

The line digraph of Complement of Cayley digraph C.Cay (G,S) is edge graceful if | S | ≡ 0(mod2).

Proof

Consider the C.Cay (G,S) with | S | = m. It has n vertices and nq arcs, where q = n-m-1. Then by the definition of the line digraph, line digraph of Complement of Cayley digraph is again a q regular digraph where q is odd. That is every vertex of the line digraph has q incoming and q outgoing arcs. If the digraph of a group contains n vertices and nq edges, then the corresponding line digraph contains nq vertices and nq^2 arcs. Let us denote the vertex set of L(C.Cay(G,S)) as V= $\{ v_1, v_2, v_3, \dots, v_{nq}\}$}and the edge set of C.Cay(G,S) as E = { eij, 1≤ i ≤ nq, 1≤ j ≤ q}.Then by the above theorem, if we define the labels for every q outgoing arcs of each vertex, we will get the edge graceful labeling of the line digraph of complement of Cayley digraph.

Antimagic labeling

A graph with q edges is called antimagic if its edges can be labeled with the set {1,2,3,…,q} such that the sums of the labels of the edges incident to each vertex are distinct.

Characterization

The complement of Cayley digraph C.Cay(G,S) with | S | ≡ 0(mod2) admits edge graceful iff it is an antimagic.

Proof

From the above theorem we know that the regular digraph with odd number of outgoing arcs is edge graceful. Now we have to prove that it is antimagic too. Define f and f* as in the above theorem.
We have,

$$f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$$

$$= i + (q-1)/2 + (n(q^2-1))/2$$

Since n and q are constants, $f^*(v_i)$ is distinct for every i, 1≤ i ≤ n. Hence the edge graceful C.Cayley digraph is always anti magic and vice versa.

Corollary

A graph G is called bi-edge - graceful if both G and its line graph L(G) are edge - graceful. Therefore the Complement of Cayley digraph C. Cay (G,S) with | S | ≡ 0(mod2) is bi - edge – graceful.

Corollary

Every cayley digraph admits edge graceful labeling only when |S| ≡ 1(mod2) but its complement is edge graceful only when |S| ≡ 0(mod2).

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Strong edge graceful labeling

Here we define a labeling called strong edge graceful labeling for digraphs by extending its range through which we can get edge graceful labeling for some family of digraphs which is not in the former case. We investigate the strong edge graceful labeling for the Complement of Cayley digraph and its line digraph.
A digraph is said to be strong edge graceful if there is an injection
f : E →{1,2,3,...,3 |E|/2} in such a way that the induced mapping $f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$ (mod 2|V|) taken over all the outgoing arcs of v_i is an injection, where e_{ij} is the j^{th} outgoing arc of the vertex v_i.

Theorem

The vertex symmetric digraph, complement of Cayley digraph C.Cay(G,S) admits strong edge graceful labeling.

Proof

Let C.Cay (G,S) be the complement of Cayley digraph with n vertices, nq arcs. Denote the vertex set of C. Cay(G,S) as V= $\{v_1, v_2, v_3, \ldots. v_n\}$ and the edge set of C.Cay(G,S) as E= $\{e_{ij}, 1 \le i \le n, 1 \le j \le q\}$ .To prove that the complement of Cayley digraph admits strong edge graceful labelling, we have to show that there exists an injection f : E →{1,2,3,...,3 |E|/2} such that the induced function $f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$ $(\text{mod } 2|V|)$ is an injection. We prove this theorem in two cases.

Case(i):

When q ≡ 1(mod2)

$$f(e_{ij}) = \begin{cases} (j-1)n + i & \text{if } j \text{ is odd} \\ jn + 1 - i & \text{if } j \text{ is even} \end{cases}$$

Then the induced function

$$f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$$

$$= i + \frac{q-1}{2} + \frac{2n(q^2-1)}{4}$$

$$= i + \frac{q-1}{2} \ (\text{mod } n)$$

Since q is constant, $f^*(v_i)$ is distinct for every i, 1≤ i ≤ n. Hence the Complement of Cayley digraph C.Cay(G,S) is strong edge graceful if q ≡ 1(mod2).

Case(ii): When q ≡ 0(mod 2)

Define f : E →{1,2,3,...,3 |E|/2} as

$$f(e_{ij}) = \begin{cases} (j-1)n+i & \text{if } j=1,3,\ldots(q-1) \text{and } q \\ jn+1-i & \text{if } j=2,4,\ldots,(q-2) \end{cases}$$

Then the induced function

$$f^*(v_i) = \sum_{j=1}^{q} f(e_{ij})$$
$$= 2i + (q-2)/2 + (q-1)n \ (\text{mod } 2n)$$

Since q is constant, $f^*(v_i)$ is distinct for every i, 1≤ i ≤ n. Hence the Complement of Cayley digraph C.Cay (G,S) is strong edge graceful.

Corollary

The line digraph of Complement of Cayley digraph C.Cay (G,S) is strong edge graceful.

Proof

Consider the C.Cay (G, S) with | S | = m generators. It has n vertices and nq arcs, where q = n-m-1. Then by the definition of the line digraph, line digraph of Complement of Cayley digraph is again a q regular digraph.

That is every vertex of the line digraph has q incoming and q outgoing arcs. If the digraph of a group contains n vertices and nq edges, then the corresponding line digraph contains nq vertices and nq2 arcs. Let us denote the vertex set of L(C.Cay(G,S)) as $V = \{v_1, v_2, v_3, \ldots, v_q\}$ and the edge set of C.Cay(G,S) as E = $\{e_{ij}, 1 \le i \le n, 1 \le j \le q\}$. Then by the above theorem, if we define the labels for every q outgoing arcs of each vertex, we will get the edge graceful labeling of the line digraph of complement of Cayley digraph.

Proposition

Every edge graceful regular digraph is strongly edge graceful. But the converse need not be true.

CONCLUSSIONS

We have investigated graceful labelings on Complement of cayley digraphs and its line digraphs. We noticed an important upshot in the above process that the number of generators of Cayley digraph plays an important role in determining the existence of labelings of the above digraph. So this offers new horizons for future work that one can examine and analyze the various other labeling on the same architectures of cayley digraphs.

CONFLICT OF INTEREST

There is no conflict of interest.

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FINANCIAL DISCLOSURE

None.

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